The Young Child and Mathematics

Juanita V. Copley

National Association for the Education of Young Children
Washington, D.C.

National Council of Teachers of Mathematics
Reston, Virginia
# Contents

*Preface*  
1. The Child Learns, the Child Teaches 1  
2. Principles in Teaching Mathematics to Young Children 13  
4. Number and Operations in the Early Childhood Curriculum 47  
5. Patterns, Functions, and Algebra in the Early Childhood Curriculum 83  
6. Geometry and Spatial Sense in the Early Childhood Curriculum 105  
7. Measurement in the Early Childhood Curriculum 125  
8. Data Analysis and Probability in the Early Childhood Curriculum 147  
9. Questions . . . and Some Answers 167  

**Appendix A**: Expectations for Pre-K–2, from NCTM’s *Principles and Standards* 177  
**Appendix B**: Making Math Meaningful through Children’s Books 180  

*References* 184  
*Index* 186
A 8-year-old with spiky hair, an impish demeanor, and a seemingly permanent pout, Timmy entered my life late one October. He was referred to me because he demonstrated few skills in reading or mathematics, seemed unmotivated to attempt new tasks, and seldom if ever completed a readable assignment.

Jessica was quiet and compliant. She never caused problems, never showed any particular gift for mathematics, never volunteered in class. In fact, I had difficulty finding anything specific to say to her parents during parent-teacher conferences.

Armand was an enthusiastic ball of energy. With hands up in the air and a grin a mile wide, Armand entered the science lab with his kindergarten class and announced, “I’m here! I love science! I can’t wait!” Within an hour I found Armand on the floor surrounded by dirt from the earthworm farm, every inch of bare skin and most of his clothing covered with dirt, holding an earthworm.

I have been a teacher for 26 years. I have taken courses, read books, earned degrees, conducted research, and published articles. I have learned mathematics, how to teach mathematics, how to assess mathematical understanding, how to understand child development, and so on. My teachers and mentors have been wonderful. They have taught me much, given me ideas, and broadened my beliefs about teaching. However, the lessons I learned from children—from Timmy, Jessica, Armand, and others—are the ones that have made the most difference in my teaching.

The Young Child and Mathematics focuses on children from ages 3 through 8 and their mathematical learning. The placement of young child before mathematics in the book’s title is not accidental. It comes first because I believe that the child should be the focus. In this chapter I share some of the lessons that I have learned about children and their understanding of mathematics. Some of the ideas come from textbooks, some from watching other teachers teach, some from experts. However, most if not all of the ideas were greatly affected by children. In some cases their strategies changed the activities for my lessons, their interests changed the scope of the content, or
their particular strengths or weaknesses changed the sequence of instruction. Whatever the case, I find that when the child is the focus of my teaching, I teach mathematics well.

**Learning from Timmy**

Let’s talk about Timmy. After looking at his work and his records, I understood his teacher’s frustration. Indeed, Timmy was a child with whom teachers found it difficult to work. He had demonstrated a very short attention span, and he showed little promise in mathematics or any other subject. I admitted him to a special program for “slow learners.”

Because of an unexpected assignment, I didn’t have time to work with Timmy that first day he appeared in my classroom. To keep him busy, I gave him a large box of electrical equipment from the sixth-grade electricity unit—switches, batteries, wire, and small light bulbs. I asked him to sort the box’s contents and told him that I would return in a while.

Thirty minutes later, I walked back into the room. Timmy had created a working electrical system! Six bulbs were lit, three switches were incorporated, and within the connection he had produced a working model of both a parallel and a series circuit. I watched and listened as this difficult, unmotivated, and unskilled 8-year-old shared with me his creation, which was much more complicated than anything I could build. Timmy explained why certain bulbs were half the brightness of other bulbs, he told me how to predict whether a bulb would light or not light, he told me about resistance and the advantages and disadvantages of series and parallel circuits.

Timmy taught me an important lesson: **Spend time observing, listening, and watching children.** Pay attention to what they like, listen to their reasoning, ask them to explain their creations, challenge them with tasks that seem impossible, and give them the opportunity to show you what they can do in the way they want.

**A lesson from Jessica**

Quiet Jessica was a child in my class during my second year of teaching. At the end of the year, I asked each child to give me a report card for being a teacher. I stressed the fact that they should tell me good things as well as things I could improve.

Jessica took my directions seriously. Her block-print note with her own spelling stated,
Ms Copley. You were real good with the dumb kids. They needed you and you helped them. You were real good with smart kids. You always kepted them busy, but you should do better with the plane kids like me. I need to learn to!

Luv, Jessica

She was right; I had spent most of my time with the special children and I had ignored the “plain” kids. I learned an important lesson with that note: Remember that every child is important! It is my job to do my best teaching for those children who have special gifts, those who need concentrated help to overcome difficulties, and those quiet, plain children who have the right to learn.

The joy of Armand

Armand’s fascination with earthworms began that first day in the science lab. He spent countless hours observing them, “reading” earthworm books in the library, and asking questions. During his study he became obsessed with finding the eyes of earthworms. Assuring me that “They gots to have eyes! They gots to see!” Armand kept asking for bigger magnifiers so that he could find their eyes.

Instead of discouraging Armand and correcting his error, I helped him set up experiments to test his hypothesis. During the last two months of the school year, Armand spent every free minute conducting experiments with colored mats, homemade earthworm houses, and colored lights. As he boarded the bus for his final trip home as a kindergartner, he yelled out the window, “I still think they gots eyes, Mrs. Copley!”

That statement taught me a great deal. Because I allowed him to investigate his hypothesis—in fact encouraged his exploration—Armand learned more about earthworms than anyone in the school. Because he had a need to know, Armand read fourth- and fifth-grade books about earthworms and could use the proper terminology to describe their parts. More important, because he was in charge of his own learning, Armand continued to be a powerful, excited learner.

When I heard Armand’s final statement, I was reminded how important it is not to correct every error. Instead, encourage investigation, and remember that children construct their own knowledge. What an important lesson for someone teaching mathematics!

These three lessons should be in evidence throughout this book. First, spend time observing, listening, and watching children. Some of the many vignettes and dialogues I have recorded, videotaped, and remembered appear in every chapter. I believe that you can learn most from children and classroom examples; thus, such examples are abundant and (using pseudonyms) presented as realistically as possible.
Second, remember that every child is important. This volume presents a variety of examples and suggested activities that work for children with all types of needs in diverse settings. Because I have been privileged to teach in multicultural settings, in urban and rural communities, in private and public schools, and in 4-year-old, kindergarten, first-, and second-grade classrooms, the ideas here reflect those contexts.

Third, encourage investigation and remember that children construct their own knowledge. I value the joy of learning, exploration, and discovery. While mathematics is often considered a subject of right answers and prescriptive instruction, the ideas presented in this book foster investigation in the way children learn.

The intuitive mathematical knowledge of the young child

Young children are natural learners. They construct their own understanding about quantity, relationships, and symbols. They approach new tasks with curiosity and a sense of experimentation. Counting is a natural task, more is a word 2-year-olds know readily, and the concepts of addition and subtraction are actively used to describe and explain situations children encounter in their world. When a new idea or piece of information doesn’t make sense to a child, Piaget theorized, the experience creates dissonance—that is, mental conflict that the child seeks to resolve. Thus the child develops and assimilates knowledge, making it his own.

The intuitive, informal mathematical knowledge of young children often surprises early childhood teachers. Yet, kindergarten curriculum tends to reflect the belief that 5-year-olds enter school as blank slates, with no concepts, no experience with quantities, patterns, shapes, or relationships!

Instead, research strongly indicates that young children have a strong, intuitive understanding of informal mathematics. To illustrate this point, listen to 3-year-old Jeffrey discuss his new set of blocks with his teacher.

Block Shapes

Ms. Wright: Tell me about your new blocks. What do you call them?

Jeffrey: Blocks with different shapes.

Ms. Wright: What can you tell me about these different shapes?

Jeffrey: This yellow one is a star. The blue one is a triangle.

Ms. Wright: Wow! Have you ever seen shapes like this before?

Jeffrey: (sighing loudly) Yeah, at my house, my Granna house, my Daddy house, outside.

Ms. Wright: You have all these shapes at everybody’s house?
Jeffrey: No . . . (picking up the orange circle) This is like my Uncle Dee’s basketball, but (frowning) it won’t bounce up and down. This is a piece of pizza (indicating a purple wedge-shape block), this is a table (a pink square), this is my best book (a rectangular block).

Jeffrey proceeds to separate the blocks into two groups—five shapes on one side and one shape on the other side. The star-shape block is alone and the others—triangle, square, pizza slice, circle, and rectangle—are heaped together in a pile.

Ms. Wright: Why did you put the star on the other side?

Jeffrey: (with a deep, long sigh that sounds as if he has lost his patience) These belong in a house and this one (the star) doesn’t. It belongs in the sky!

Did you notice Jeffrey’s verbal labeling of the block shapes, his connection of the blocks to items he regularly sees, and his clarification of differences between the basketball and the round block? While his particular classification system (belongs in the house, doesn’t belong in the house) is not found in state objectives, his system is perfectly correct and indicates a consistency not typical in such a young child.

Identification of two-dimensional shapes is an objective in kindergarten programs; Jeffrey has already shown that ability at age 3. The identification of shapes in the everyday world is also a standard objective in kindergarten curricula; again Jeffrey demonstrates that ability. I am not proposing that all 3-year-olds have Jeffrey’s understanding and use of language; however, I do believe that many early childhood programs and teachers view young children as incapable, when in fact they already grasp many mathematical concepts at an intuitive level.

While reviewing research studies is not the purpose of this chapter, I have listed below some important points supported by research that illustrate the intuitive mathematical knowledge of the young child.

- Young children have a wealth of informal knowledge about mathematics as a result of everyday experiences and strategies they create to deal with events in their lives.
- The operations of addition, subtraction, multiplication, and division are often understood by young children. While they may not be able to complete a written equation like 5 – 2 = 3, they can easily tell you how many buttons you would have on your shirt if you started with five and two fell off. Accordingly, they can figure out how many pieces of candy to purchase for a birthday party if everyone attending got three pieces.
- Children’s understanding of rational numbers, while incomplete, is often more accomplished than expected. Their common sharing experiences, their use of the term half, and their fair distribution of quantities among friends are natural by-products of everyday experiences.
The development of geometric concepts and spatial sense can often be observed when young children participate in free play. The young child directing a building block project uses words and motions to tell his friend how to make a castle. “Do it like me. You need a square block” (his description for a cube). “No, not that way. Turn it over!” When his friend says, “It doesn’t matter, it’s always the same,” the child reaffirms his own understanding by saying, “Well that’s ‘cause now it’s right!” Both boys are experimenting with beginning concepts of rotation and the language of geometry.

A natural fascination with large numbers is evident with young children. While they frequently invent nonsensical numbers like “a million, dillion, kilion,” they often show a partial understanding of quantity and the need for counting.

Do we need to directly teach young children all they need to know about mathematics? Do we need to start from the beginning, drill in those basic facts, and fill all the holes in their understanding? Do we need to tightly define developmentally appropriate mathematics as easy concepts all children can learn?

The answer to all of these questions is a resounding No! Instead, we need to remember that young children possess a vast amount of intuitive, informal mathematical knowledge. Our job is to assess their prior knowledge, build upon their strengths, facilitate their learning, and enjoy the process.

The constructed mathematical concepts of the young child

Young children continually construct mathematical ideas based on their experiences with their environment, their interactions with adults and other children, and their daily observations. These constructed ideas are unique to each child and vary greatly among children the same age. Some of the ideas are perceptually immature, many of the problem-solving strategies are inefficient, and the verbal information necessary to discuss mathematics is often incorrectly labeled or modeled. The child who perceives more candy to be in the larger bag, the child who always adds together two sets of items using the counting-all strategy, and the child who counts, “6, 7, 8, 9, 10, oneteen, twoteen, threeteen,” illustrate mathematical ideas that would be labeled incorrect for an adult but are often developmentally correct for a young child.

An early childhood teacher who frequently listens to ideas expressed by young children can provide materials and an environment conducive to the development of mathematical concepts. More important, by observing young children the teacher can ask questions that prompt them to make new discoveries and form their own questions. To illustrate this point, listen to some prekindergartners’ responses to a problem presented by their classroom teacher.
Six Legs, Five Fingers

Miss Riley: I just bought this bee puppet (shows a bee puppet attached to a black six-fingered glove) and I have a problem. When I put my hand into the glove, I always have an empty leg on the puppet. I don’t have enough fingers for all of the legs. I have only five fingers and there are six legs on this puppet. I don’t know what to do! Maybe I’ll just take it back to the store.

The children seem to be thinking about this problem. Some count their fingers, others talk to their partners, and still others shrug their shoulders.

Russell: Well, maybe you got your fingers in wrong.

Miss Riley: Maybe. Let’s see. (The teacher puts her hand in and out of the puppet a few times, each time showing the leg without the finger.) No, it’s still there.

Marta: Just cut it off.

Miss Riley: Well, that’s an idea, but it’s brand new. I don’t want to cut up a brand new puppet!

Svetlana: I know. Let me show you. (Svetlana tries to put one of the legs inside the glove. It leaves a hole, and Svetlana shrugs and sits down.)

Miss Riley: Good try, Svetlana, but it still seems like it’s not quite right.

Duane: Hey, maybe the guy who made it had six fingers!

This idea seems to satisfy many of the children, who nod. The children are excited about a possible solution.

Miss Riley: Maybe so. Let’s see. How many fingers do you have on one hand?

Everyone spends time counting and recounting the fingers on their hands. Some repeat the counting three or four times.

Miss Riley: Does anyone have six fingers on a hand?

The children shake their heads no or respond verbally, saying they have five fingers. Others act like they do have six fingers, count aloud as they touch them, and then say that they have only five. After a few minutes they seem to be satisfied that no one in the class has six fingers.

Linda: (excitedly raising her hand) Umm . . . umm . . . maybe bees have six legs!

Duton: No. I have lots of bees at my house and they all have five legs! (Linda looks disappointed; because Duton is a class leader, most students believe he is right.)

Miss Riley: Maybe Linda has a good idea. How could we find out?