Mathematical Pattern Hunters

Phyllis Whitin and David J. Whitin

The habit of looking for patterns, the skills to find them, and the expectation that patterns have explanations" is an essential mathematical habit of mind for young children (Goldenberg, Shteingold, & Feurzeig 2003, 23). Work with patterns leads to the ability to form generalizations, the bedrock of algebraic thinking, and teachers must nurture this skill in children in the early childhood years (NCTM 2000; NAEYC & NCTM 2002).

In this article we look at children's mathematical investigations of patterns inspired by the book *One Is a Snail, Ten Is a Crab*, by April Pulley Sayre and Jeff Sayre. We describe how the children in a second and third grade mixed-age classroom used talk, writing, drawing, and gestures to represent their growing understanding of numerical patterns (Dacey & Eston 1999; Trafton & Thiessen 1999; Whitin & Whitin 2000).

Setting the stage with a counting book

One Is a Snail, Ten Is a Crab is an entertaining picture book that features familiar animals with different numbers of feet enjoying a day at the beach. Cartoon-style illustrations add humor and appeal, inviting young readers to identify combinations of legs that equal 1 to 10 and then sets of 10 to 100. Even numbers between 1 and 10 feature a single animal—6 for an insect and 8 for a spider—with the illustrations accompanied by large arrows pointing to the legs. Odd numbers add a snail—"9 is a spider and a snail." Pages of multiples of 10 show two examples: "30 is 3 crabs . . . or 10 people and a crab."

The book offered several benefits for the second- and third-graders with whom we two authors, Phyllis and David, were working: it highlights relationships between combinations for 10, patterns of odd and even numbers, and patterns involving a set model for multiplication (Whitin & Whitin 2009). When we read the book aloud, the children commented about patterns they observed. They noticed the odd-even pattern, chiming in that "9 is a spider and a snail" after seeing that 3 was "a person and a snail," 5 was "a dog and a snail," and 7 was "an insect and a snail." Jackson saw another alternating pattern using crabs for multiples of 10: "40 is 10 dogs, but 50 is 10 dogs and a crab; 60 is 10 insects, and 70 is 10 insects and a crab."

After the children discussed these patterns, we invited them to create their own 10s books. Like *One Is a Snail*, the children's books featured sets of animals' legs representing multiples of 10—spiders and people (8 + 2), butterflies and cats (6 + 4), and starfish (5 + 5). The children labeled the pages as consecutive multiples of ten (10, 20, . . . 50), and they represented their understandings of each number on its page. During the process of creating, sharing, and examining numerical relationships in their books, the children decided to call themselves "pattern hunters."

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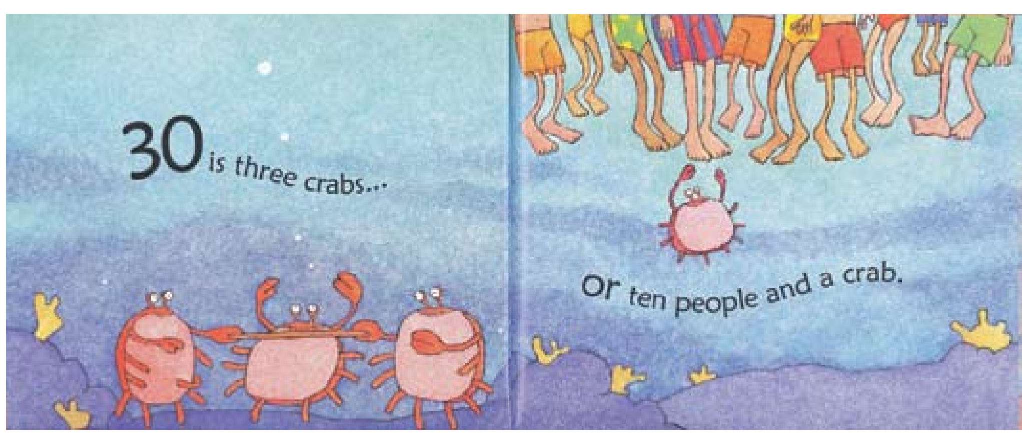
A study guide for this article is available online at **www.naeyc.org/yc**.



Finding patterns in the children's 10s books

Aimee began her book by drawing 2 deer and 1 person to make 10. As she continued her work to show 20 and 30, she discovered a pattern that highlighted how sets of 2 (people's legs) are related to sets of 4 (deer's legs). She first drew 4 deer for 20 and counted the 16 legs. Next she tapped her paper to count as she continued aloud: "Seventeen, 18, 19, 20. I need one more deer!" She was surprised to discover that the legs of 5 deer equal 20 and she did not need a person. For 30, she drew 7 deer (28) and a person (2). She showed her work to Phyllis, who remarked, "Interesting! Let's look at what you have so far."

Phyllis suggested that Aimee continue to investigate this pattern, commenting, "Mathematicians make predictions and test them out."

Phyllis spread out Aimee's pages for 10, 20, and 30 so they could examine the sequence (10 = 2 deer and 1 person, 20 = 5 deer and no person, 30 = 7 deer and 1 person). Aimee exclaimed, "It's a pattern!" She pointed out that every other page in her book "needs a person." Phyllis suggested that Aimee continue to investigate this pattern, commenting, "Mathematicians make predictions and test them out."

Children express patterns

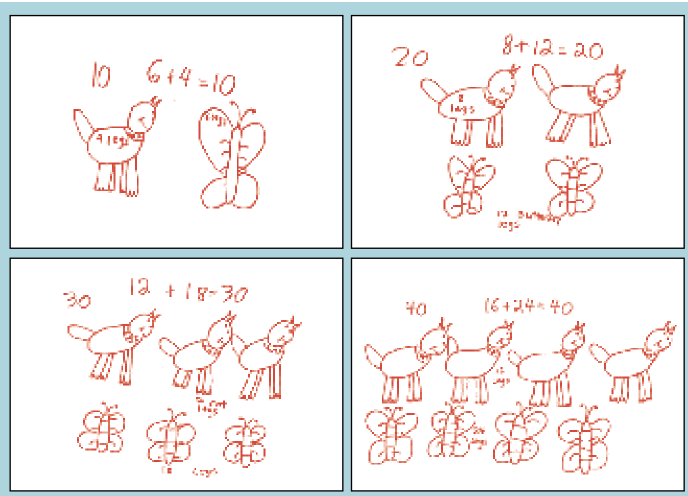
The next day, we gathered the children together to examine one of the child-authored books and talk about the patterns they could see. In her book, Mei showed increasing sets of butterfly legs (6) and dogs' legs (4), highlighting the 6 + 4 combinations to make sets of 10 (see "Mei's Sets of 10"). The children noticed many patterns in her work:

- "She added one more [of each animal] each time."
- "It went 2, 4, 6, 8 [the total number of animals on each page]."
- " It's easy to count them up [because of the pairing of 6s with 4s]."
- "I know the pattern. The last page is going to be 4 dogs and 4 butterflies for 40."

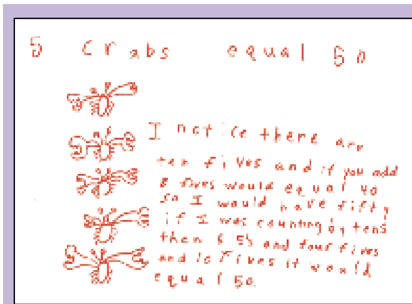
Inviting the children to observe a peer's work gave them another context in which to investigate numerical relationships, and their insightful comments confirmed the children as both pattern hunters and pattern creators. Following this warm-up activity, the children continued working on their books.

Cameron noticed an interesting pattern as he worked. He wrote, "2 crabs equal 20, and 1 crab and an insect and a dog make 20 too. 3 crabs equal 30, and 2 crabs and an insect and a dog make 30 too." Thus, he created alternative names for his sets of 10 by reducing the number of crabs by 1 and then substituting for the missing crab an equivalent—an insect and a dog (6 + 4 = 10). We can represent Cameron's numerical pattern like this:

20 2 crabs or 1 crab + 1 insect and 1 dog
30 3 crabs or 2 crabs + 1 insect and 1 dog
40 4 crabs or 3 crabs + 1 insect and 1 dog



Mei's sets of 10

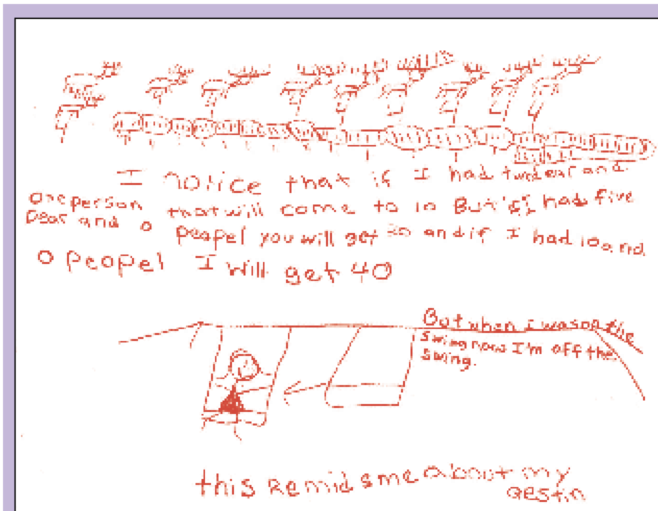


Cameron's 50 discovery

Cameron found another pattern that highlighted the relationship between 5 and 10 when he created his page for 50 (see "Cameron's 50 Discovery"). He explained to David, "There's 5 tens [crabs], that's 50. And there are 10 fives [each crab had two sets of 5 legs], and that's 50 too." Cameron then discussed how 40 equals 4 tens or 8 fives, and 30 equals 3 tens or 6 fives. David suggested making a chart to show this relationship (see "Cameron's Ratio Table").

Meanwhile, Aimee, now working on the 40 page in her deer-and-people book, found that her pattern of "needing a person" and "not needing a person" continued—40 *did* equal 10 deer and no person. Phyllis encouraged Aimee to write about the pattern she had discovered. Aimee began her sentence with "I notice," reflecting the language we adults had consistently modeled in our discussions: "I notice that if I had two deer and one person that will come to 10. But if I had five deer and 0 people you will get to 20 and if I had 10 and 0 people I will get 40."

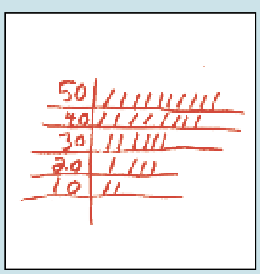
Phyllis continued, "What does your pattern remind you of?" Like Cameron, Aimee used a personal context to describe her pattern. She drew 2 swings, one occupied and one empty, and explained that sometimes two friends were on the swings together and one friend jumped off. Her connection and representation nicely captured the alternating pattern of sets of 4 (deer only) and sets of 4s and 2s (deer and people).

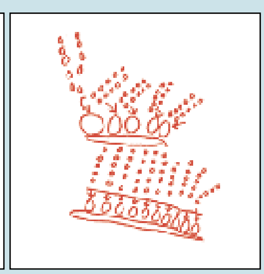


Aimee's 40 page

Cameron's Ratio Table

After Cameron's discovery of the relationship between 5s and 10s, David helped him begin a chart to scaffold his mathematical understanding. David listed the multiples of 10, and Cameron then used tally marks to represent the 5s. The chart represented a ratio table: each crab is composed of two sets of 5, so for every multiple of 10 there are twice the number of 5s. Ratios compare two quantities or measures, and the chart helps to highlight this patterned relationship.





Pointing to the chart, David said, "As you noticed earlier, your paper shows 10 fives make 50, but also 5 tens make 50. What does that remind you of?" (This question is helpful because it challenges children to make a personal connection to the pattern.) Cameron drew a picture and explained his connection this way: "It's like 5 pieces of chicken, each cut into 10 pieces. And 10 pieces of chicken each cut into 5 pieces." He emphasized his analogy by making a cutting motion in the air with his hand. His gestures, his drawing, and his chart all contributed to his representation of this numerical pattern.

Next, Phyllis mused, "I wonder what you might predict for 80." Aimee wasn't sure. Using her pictures of deer, she decided to count the 40 legs twice to get to 80. At first as she counted, she lost her place, so she devised a strategy of making sets of 4 tick marks (below her illustrations of deer) to systematically keep track and verify her findings. Finishing the last set, she confidently declared that for 80, "you don't need a person."

Anthony, who sat nearby, had apparently been listening to the conversation. He piped up, "Hey, you could just double the number for people." He had discovered yet another pattern. Phyllis asked Anthony to elaborate. He said, "You need 10 deer for 40 legs, and each deer is like 2 people, so you'd need 20 people for 40 legs," His observation showed the 1:2 ratio between corresponding sets of 2 and 4. The threesome then examined this relationship as well. Pattern hunters are collaborators!

We used the opportunity to work individually with a few children, and then we gave them the responsibility of mentoring some of their peers.

Extending the investigation

We next gave the children some problems to solve that were based on our informal individual conversations with them while they were working on their 10s books. By asking questions related to their books, we wanted not only to honor the children's work but also to give them further experience in using the model of sets for multiplication (the deer problem) and using combinations for 10 (spiders-and-people problem).

One problem, two solutions

Aimee's work with the deer-and-people combinations to make 10 prompted us to pose this problem for the class to solve:

Can you show 40 legs using only deer?

The children revealed different patterns as they addressed the problem in various ways. Isaiah, for instance, counted on by sets of 4 and wrote, "4, 8, 12, 16, 20, 24, 28, 32, 36, 40. Deer have 4 legs, so count by 4 ten times, and you have 40 legs." His list of multiples of 4 highlights the pattern of multiplication as a set that is repeated over and over. Jackson drew only one deer and wrote "4 x 10 = 40" below it, remarking, "I just know 4 times 10 is 40."

Ashley identified still another pattern. She drew 5 circles to represent 5 deer and then counted by 4s to find that 5 deer equal 20 legs. She then realized that she was halfway to 40 legs, and wrote, "All you have to do is double!" Ashley knew that if she doubled the number of deer, she would also be doubling the number of legs, and then she would have a total of 10 deer with 40 legs. She wrote "4 x 10 = 40" to summarize her work. Finding half the total and then doubling it is a useful pattern for determining the total. (For another way to express Ashley's work, see "Multiplication and the Associative Property.")

Representing patterns in many ways

Maria's work with spiders and people provided the basis for our second problem:

Continue this pattern. How many spiders and people will equal 30 legs? 40 legs? 50 legs?

Spiders	People	Legs
1	1	8 + 2 = 10
2	2	8 + 8 + 2 + 2 = 20

As the children worked on the spider-and-people-pattern problem, we anticipated that some would have difficulty adding strings of 8s and strings of 2s together. We used the opportunity to work individually with a few children, and then we gave them the responsibility of mentoring some of their peers.

Devising a strategy. David came by Kayla's desk as she began completing the chart of spiders and people. She had written down a series of 8s and a series of 2s next to each line of the pattern, and now she had to add up the total. She began with "8 + 8 + 8 + 2 + 2 + 2 = ?" David asked her how she was going to add those numbers, and she proceeded to add them in the order in which she had written them.

David asked Kayla if she could think of another way, but Kayla was not sure. He suggested, "What if we hooked up this 8 with a 2, what would we find?" "Oh, that's a 10," she replied. Then, pausing in thought for a second, she exclaimed, "Hey, you could do that with all of them!" "So how could you show on your paper that you hooked up this 8 with this 2?" asked David. Kayla answered, "I could just draw some loops, like this" (see "Kayla's Looping Strategy," p. 88).With much excitement, she began to draw loops to connect each 8 with its corresponding 2. She then added together her 10s and recorded all her answers.

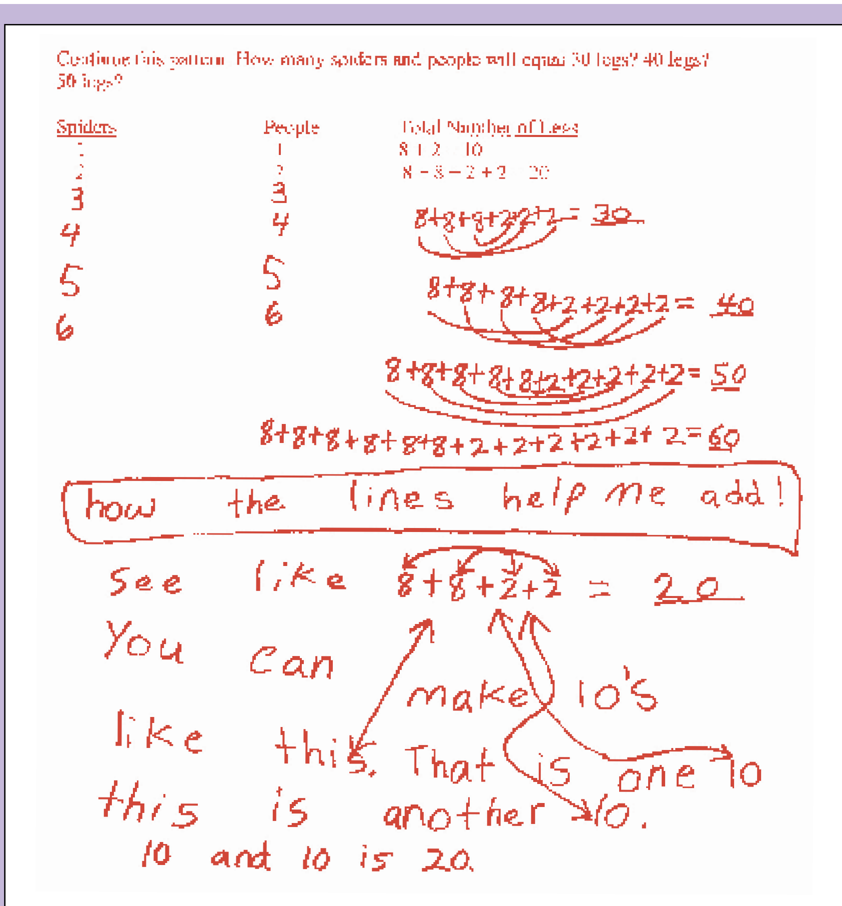
Kayla was so excited about the looping strategy that she created a problem for her friends to solve: "Continue this pattern with bees and dogs." She then started a chart of adding together sets of bees and dogs: "6 + 4 =?, 6 + 6 + 4 + 4 = ?" and so on. She of course solved it herself by

Multiplication and the Associative Property

In hindsight, we could have shown Ashley another way to record what she did. Since she divided the 10 deer into two sets of 5 (by doubling the five circles), she could have renamed the 10 as 2 x 5. She then multiplied the 5 deer by 4 legs each to get 20 and then doubled that 20 to get to 40. We could have guided Ashley to explain it mathematically in this way:

4 x 10	Each of 10 deer have 4 legs
4 x (2 x 5)	You renamed the set of 10 deer as 2 sets of 5 deer
(4 x 5) x 2	You found how many legs are in one set of 5 deer
20 x 2	And then you multiplied that answer by 2 because you have 2 sets of 5 deer each

Ashley's example is a good demonstration of how the *associative property* works—that is, in multiplying three or more numbers, the order of the numbers does not matter.



Kayla's looping strategy

drawing loops to match each 6 with a 4 to make "easy 10s." After sharing this strategy with several of her classmates, Kayla became known as the pattern hunter who specialized in finding 10s.

Explaining strategies through personal connections. Kayla's friend Victoria liked the looping strategy because when adding, she too had not been flexible in looking for easy combinations. Victoria made a personal connection to Kayla's idea by linking it to her scientific knowledge.

Victoria often talked about her two birds at home that mate and lay eggs. She used mating as an analogy to describe the process of making 10s: when she hooked each of the 8s in a string of 8s with a 2 in the string of 2s, she said, "Each 8 needs a 2 for a mate, which equals 10." She used this personal analogy to explain this newly discovered looping pattern.

Connecting symbols with objects. Kennedy became so consumed with the task of adding the strings of 8s and 2s that she lost track of what the numbers represented. Phyllis asked her to put aside her paper and think about the spiders and the people. She felt that focusing on the context of the animals' legs would enable Kennedy to make sense of the problem without being distracted by the symbols on the page. Kennedy easily answered the questions that Phyllis posed: "Where does the first 8 come from?" [1 spider], "Where does the first 2 come from?" [1 person], and "How many legs do they have together?" [10].

Phyllis continued, "Let's pretend that the spider and the person walked through the door together. That's 10 legs. What if another spider and person came through the door?" "Twenty!" Kennedy exclaimed. She imagined successive spider-and-person sets coming through the door to total 30, 40, and 50 legs. Now the strings of numbers made more sense. "Think of putting together a spider and a person each time," Phyllis reminded her.

Phyllis then invited Kayla, who had just learned the looping strategy, to share her newfound knowledge with Kennedy, who quickly caught on. Picturing sets of spiders and people coming through the door helped Kennedy see patterns and use those patterns to transition from the concrete examples in the book to abstract calculations on paper.

Eliciting patterns through drawings. Cameron used both pictures and numbers to record his results, thereby displaying some additional patterns. He drew boxes around each set of creatures to be added together. By placing the creatures on top of each other and labeling each set with the same number (for example, in the 30 box, 3 spiders, 3 people), he emphasized not only the equivalent number of sets for each of the creatures, but also the equivalence of these sets with the total number of 10s (3 sets of spiders and 3 sets of people equal 3 sets of 10). Thus, drawings can sometimes illuminate patterns that cannot be

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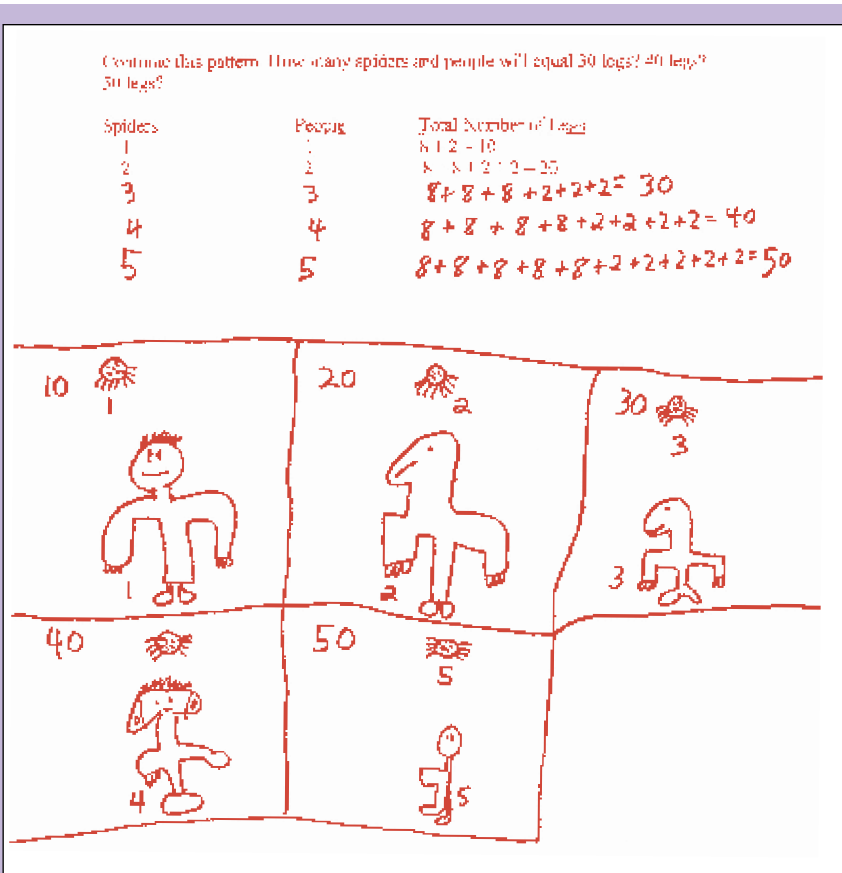

Cameron's solution

"Can you show me where these numbers came from?" she asked, pointing to the 24. He immediately explained that the 24 came from the 3 spiders (3 x 8 = 24). Phyllis suggested that Hassan write down this information so that his classmates could understand his important mathematical strategy. Hassan then wrote independently, "The 32 came from 4 spiders and the 8 came from [4] people [4 x 2 = 8]. The 40 came from the 5 spiders and the 10 from [5] people [for 50]. The 48 came from the 6 spiders and the 12 from the people [6 x 2 = 12]."

Each of Hassan's representations, the numerical chart and the written commentary, reveals important relationships. Because he organized his chart by following a consistent form of spiders + people, the patterns among multiples are easy to recognize. By looking at the vertical columns, several patterns emerge: 24, 32, 40, and 48 increase by 8s, while 6, 8, 10, and 12 increase by 2s. The totals not only increase by 10 but also represent the 10 combination of 8 + 2. Hassan's narrative justifies his use of multiplication as an efficient and accurate

seen as clearly through a chart of numbers. Cameron's drawings and Kayla's loops helped to show this increasing pattern of 10s in different ways.

Expressing understanding in numbers, operations, and writing. Hassan, confident and proficient in his ability to calculate, sometimes had difficulty expressing his process of reasoning. He realized that multiplication was more efficient than addition to find totals of sets of legs. Instead of adding the strings of 8s and 2s, he wrote:

24 + 6 = 30
32 + 8 = 40
40 + 10 = 50
48 + 12 = 60

Phyllis was impressed, but she also wanted Hassan to justify his work.

way to calculate sets of like numbers. Hassan's experience, along with Kayla's, Kennedy's, and Cameron's, shows that pattern hunters represent their understanding in many ways.

Building a community of pattern hunters

As we reflected on this experience, we identified some important ways teachers can nurture in children a disposition for pattern hunting. Here are a few of those ways.

• Share quality math-related children's literature that highlights patterns (see "Good Books for Exploring Patterns in Mathematics"). Books provide an authentic context for mathematical patterns and relationships and can be the catalyst for meaningful discussions and explorations (Whitin & Whitin 2004).

• Pose questions that invite children to observe closely: What patterns do you notice? What do you predict? How can you show this pattern?

• Encourage children to represent patterns in different ways. Creating drawings, using arrows, making connecting loops, and organizing a chart are just some of the ways that these second- and third-grade children represented their understandings. Often one form of representation reveals a pattern more clearly than another.

Thus, through the use of good books, careful questioning, and varied opportunities for expression, teachers increase avenues for understanding, thereby making it more likely that all children will be successful in mathematics.

Good Books for Exploring Patterns in Mathematics

Aker, Suzanne. 1990. *What Comes in 2's, 3's, & 4's?* New York: Simon & Schuster.
Anno, Mitsumasa. 1982. *Anno's Counting House.* New York: Philomel.
Appelt, Kathi. 1999. *Bats on Parade*. New York: HarperCollins.
Cuyler, Margery. 2000. *100th Day Worries.* New York: Simon & Schuster.
Dee, Ruby. 1988. *Two Ways to Count to Ten: A Liberian Folktale.* New York: Henry Holt.
DeFelice, Cynthia. 2006. *One Potato, Two Potato.* New York: Farrar, Straus and Giroux.
Giganti, Paul. 1992. *Each Orange Had 8 Slices.* New York: Scholastic.
Guettier, Bénédicte. 1999. *The Father Who Had 10 Children.* New York: Dial.
Hamm, Diane Johnston. 1991. *How Many Feet in the Bed?* New York: Simon & Schuster.
Harris, Trudy. 2000. *Pattern Fish.* Brookfield, CT: Millbrook.
Hong, Lily. 1993. *Two of Everything.* Morton Grove, IL: Albert Whitman.
Hulme, Joy N. 1991. *Sea Squares.* New York: Hyperion.
Hutchins, Pat. 1994. *The Doorbell Rang.* New York: Greenwillow.
Hutchins, Pat. 2002. *Ten Red Apples.* New York: Greenwillow.
Jenkins, Emily. 2001. *Five Creatures*. New York: Farrar, Straus and Giroux.
MacDonald, Susi. 2000. *Look Whooo's Counting.* New York: Scholastic.
Merriam, Eve. 1993. *12 Ways to Get to 11.* New York: Simon & Schuster.
Pinczes, Elinor J. 1993. *One Hundred Hungry Ants.* Boston: Houghton Mifflin.
Pinczes, Elinor J. 1995. *A Remainder of One.* Boston: Houghton Mifflin.
Ross, Tony. 2002. *Centipede's 100 Shoes.* New York: Henry Holt.
Sturges, Philemon.1995. *Ten Flashing Fireflies.* New York: North-South Books.
Tang, Greg. 2001. *The Grapes of Math: Mind-Stretching Math Riddles.* New York: Scholastic.
Wells, Rosemary. 2000. *Emily's First 100 Days of School.* New York: Hyperion.

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NCTM. 2000. *Principles and Standards for School Mathematics*. Reston, VA: Author. http://standards.nctm.org/documents/index.htm.
Trafton, P., & D. Thiessen. 1999. *Learning through Problems: Number Sense and Computational Strategies*. Portsmouth, NH: Heinemann.
Whitin, D.J., & P. Whitin. 2004. *New visions for linking literature and mathematics*. Urbana, IL: National Council of Teachers of English (NCTE); Reston, VA: NCTM.
Whitin, D.J., & P. Whitin. 2009. "Legs and More Legs." *Teaching Children Mathematics* 16 (2): 80–87.
Whitin, P., & D.J. Whitin. 2000. *Math Is Language Too: Talking and Writing in the Elementary School Classroom*. Urbana, IL: NCTE; Reston, VA: NCTM

Books provide an authentic context for mathematical patterns and relationships and can be the catalyst for meaningful discussions and explorations.