naeyc®

Early Childhood Mathematics: Promoting Good Beginnings

A joint position statement of the National Association for the Education of Young Children (NAEYC) and the National Council of Teachers of Mathematics (NCTM). Adopted in 2002. Updated in 2010.

Position

The National Council of Teachers of Mathematics (NCTM) and the National Association for the Education of Young Children (NAEYC) affirm that high-quality, challenging, and accessible mathematics education for 3- to 6-year-old children is a vital foundation for future mathematics learning. In every early childhood setting, children should experience effective, research-based curriculum and teaching practices. Such high-quality classroom practice requires policies, organizational supports, and adequate resources that enable teachers to do this challenging and important work.

The challenges

Throughout the early years of life, children notice and explore mathematical dimensions of their world. They compare quantities, find patterns, navigate in space, and grapple with real problems such as balancing a tall block building or sharing a bowl of crackers fairly with a playmate. Mathematics helps children make sense of their world outside of school and helps them construct a solid foundation for success in school. In elementary and middle school, children need mathematical understanding and skills not only in math courses but also in science, social studies, and other subjects. In high school, students need mathematical proficiency to succeed in course work that provides a gateway to technological literacy and higher education [1–4]. Once out of school, all adults need a broad range of basic mathematical understanding to make informed decisions in their jobs, households, communities, and civic lives.

Besides ensuring a sound mathematical foundation for all members of our society, the nation also needs to prepare increasing numbers of young people for work that requires a higher proficiency level [5, 6]. The National Commission on Mathematics and Science Teaching for the 21st Century (known as the Glenn Commission) asks this question: “As our children move toward the day when their decisions will be the ones shaping a new America, will they be equipped with the mathematical and scientific tools needed to meet those challenges and capitalize on those opportunities?” [7, p. 6]

Since the 1970s a series of assessments of U.S. students' performance has revealed an overall level of mathematical proficiency well below what is desired and needed [5, 8, 9]. In recent years NCTM and others have addressed these challenges with new standards and other resources to improve mathematics education, and progress has been made at the elementary and middle school levels—especially in schools that have instituted reforms [e.g., 10–12]. Yet achievement in mathematics and other areas varies widely from state to state [13] and from school district to school district. There are many encouraging indicators of success but also areas of continuing concern. In mathematics as in literacy, children who live in poverty and who are members of linguistic and ethnic minority groups demonstrate significantly lower levels of achievement [14–17].

If progress in improving the mathematics proficiency of Americans is to continue, much greater attention must be given to early mathematics experiences. Such increased awareness and effort recently have occurred with respect to early foundations of literacy. Similarly, increased energy, time, and wide-scale commitment to the early years will generate significant progress in mathematics learning.

The opportunity is clear: Millions of young children are in child care or other early education settings where they can have significant early mathematical experiences. Accumulating research on children's capacities and learning in the first six years of life confirms that early experiences have long-lasting outcomes [14, 18]. Although our knowledge is still far from complete, we now have a fuller picture of the mathematics young children are able to acquire and the practices to promote their understanding. This knowledge, however, is not yet in the hands of most early childhood teachers in a form to effectively guide their teaching. It is not surprising then that a great many early childhood programs have a considerable distance to go to achieve high-quality mathematics education for children age 3-6.

In 2000, with the growing evidence that the early years significantly affect mathematics learning and attitudes, NCTM for the first time included the prekindergarten year in its Principles and Standards for School Mathematics (PSSM) [19]. Guided by six overarching principles—regarding equity, curriculum, teaching, learning, assessment, and technology—PSSM describes for each mathematics content and process area what children should be able to do from prekindergarten through second grade.

NCTM Principles for School Mathematics

Equity: Excellence in mathematics education requires equally high expectations and strong support for all students.

Curriculum: A curriculum is more than a collection of activities; it must be coherent, focused on important mathematics, and well articulated across the grades.

Teaching: Effective mathematics teaching requires understanding of what students know and need to learn and then challenging and supporting them to learn it well.

Learning: Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge.

Assessment: Assessment should support the learning of important mathematics and furnish useful information to both teachers and students.

Technology: Technology is essential to teaching and learning mathematics; it influences the mathematics that is taught and enhances students' learning.

Note: These principles are relevant across all grade levels, including early childhood.

The present statement focuses on children over 3, in large part because the knowledge base on mathematical learning is more robust for this age group. Available evidence, however, indicates that children under 3 enjoy and benefit from various kinds of mathematical explorations and experiences. With respect to mathematics education beyond age 6, the recommendations on classroom practice presented here remain relevant. Further, closely connecting curriculum and teaching for children age 3–6 with what is done with students over 6 is essential to achieve the seamless mathematics education that children need.

Recognition of the importance of good beginnings, shared by NCTM and NAEYC, underlies this joint position statement. The statement describes what constitutes high-quality mathematics education for children 3–6 and what is necessary to achieve such quality. To help achieve this goal, the position statement sets forth 10 research-based, essential recommendations to guide classroom[1] practice, as well as four recommendations for policies, systems changes, and other actions needed to support these practices.

In high-quality mathematics education for 3- to 6-year-old children, teachers and other key professionals should

1. enhance children's natural interest in mathematics and their disposition to use it to make sense of their physical and social worlds
2. build on children's experience and knowledge, including their family, linguistic, cultural, and community backgrounds; their individual approaches to learning; and their informal knowledge
3. base mathematics curriculum and teaching practices on knowledge of young children's cognitive, linguistic, physical, and social-emotional development
4. use curriculum and teaching practices that strengthen children's problem-solving and reasoning processes as well as representing, communicating, and connecting mathematical ideas
5. ensure that the curriculum is coherent and compatible with known relationships and sequences of important mathematical ideas
6. provide for children's deep and sustained interaction with key mathematical ideas
7. integrate mathematics with other activities and other activities with mathematics
8. provide ample time, materials, and teacher support for children to engage in play, a context in which they explore and manipulate mathematical ideas with keen interest
9. actively introduce mathematical concepts, methods, and language through a range of appropriate experiences and teaching strategies
10. support children's learning by thoughtfully and continually assessing all children's mathematical knowledge, skills, and strategies.

To support high quality mathematics education, institutions, program developers, and policy makers should

1. create more effective early childhood teacher preparation and continuing professional development
2. use collaborative processes to develop well aligned systems of appropriate high-quality standards, curriculum, and assessment
3. design institutional structures and policies that support teachers' ongoing learning, teamwork, and planning
4. provide resources necessary to overcome the barriers to young children's mathematical proficiency at the classroom, community, institutional, and system-wide levels.

[1] *Classroom* refers to any group setting for 3- to 6-year-olds (e.g., child care program, family child care, preschool, or public school classroom).

Recommendations

Within the classroom

To achieve high-quality mathematics education for 3- to 6-year-old children, teachers[2] and other key professionals should

1. **Enhance children's natural interest in mathematics and their disposition to use it to make sense of their physical and social worlds.**

Young children show a natural interest in and enjoyment of mathematics. Research evidence indicates that long before entering school children spontaneously explore and use mathematics—at least the intuitive beginnings—and their mathematical knowledge can be quite complex and sophisticated [20]. In play and daily activities, children often explore mathematical ideas and processes; for example, they sort and classify, compare quantities, and notice shapes and patterns [21–27].

Mathematics helps children make sense of the physical and social worlds around them, and children are naturally inclined to use mathematics in this way ("He has more than I do!" "That won't fit in there—it's too big"). By capitalizing on such moments and by carefully planning a variety of experiences with mathematical ideas in mind, teachers cultivate and extend children's mathematical sense and interest.

Because young children's experiences fundamentally shape their attitude toward mathematics, an engaging and encouraging climate for children's early encounters with mathematics is important [19]. It is vital for young children to develop confidence in their ability to understand and use mathematics—in other words, to see mathematics as within their reach. In addition, positive experiences with using mathematics to solve problems help children to develop dispositions such as curiosity, imagination, flexibility, inventiveness, and persistence that contribute to their future success in and out of school [28].

[2] *Teachers* refers to adults who care for and educate groups of young children.

2. **Build on children's experience and knowledge, including their family, linguistic, cultural, and community backgrounds; their individual approaches to learning; and their informal knowledge.**

Recognizing and building on children's individual experiences and knowledge are central to effective early childhood mathematics education [e.g., 20, 22, 29, 30]. While striking similarities are evident in the mathematical issues that interest children of different backgrounds [31], it is also true that young children have varying cultural, linguistic, home, and community experiences on which to build mathematics learning [16, 32]. For example, number naming is regular in Asian languages such as Korean (the Korean word for "eleven" is *ship ill*, or "ten one"), while English uses the irregular word *eleven*. This difference appears to make it easier for Korean children to learn or construct certain numerical concepts [33, 34]. To achieve equity and educational effectiveness, teachers must know as much as they can about such differences and work to build bridges between children's varying experiences and new learning [35–37].

In mathematics, as in any knowledge domain, learners benefit from having a variety of ways to understand a given concept [5, 14]. Building on children's individual strengths and learning styles makes mathematics curriculum and instruction more effective. For example, some children learn especially well when instructional materials and strategies use geometry to convey number concepts [38].

Children's confidence, competence, and interest in mathematics flourish when new experiences are meaningful and connected with their prior knowledge and experience [19, 39]. At first, young children's understanding of a mathematical concept is only intuitive. Lack of explicit concepts sometimes prevents the child from making full use of prior knowledge and connecting it to school mathematics. Therefore, teachers need to find out what young children already understand and help them begin to understand these things mathematically. From ages 3 through 6, children need many experiences that call on them to relate their knowledge to the vocabulary and conceptual frameworks of mathematics—in other words, to “mathematize” what they intuitively grasp. Toward this end, effective early childhood programs provide many such opportunities for children to represent, reinvent, reorganize, quantify, abstract, generalize, and refine that which they grasp at an experiential or intuitive level [28].

3. Base mathematics curriculum and teaching practices on knowledge of young children’s cognitive, linguistic, physical, and social-emotional development.

All decisions regarding mathematics curriculum and teaching practices should be grounded in knowledge of children’s development and learning across all interrelated areas—cognitive, linguistic, physical, and social-emotional. First, teachers need broad knowledge of children’s cognitive development—concept development, reasoning, and problem solving, for instance—as well as their acquisition of particular mathematical skills and concepts. Although children display mathematical ideas at early ages [e.g., 40–43] their ideas are often very different from those of adults [e.g., 26, 44]. For example, young children tend to believe that a long line of pennies has more coins than a shorter line with the same number.

Beyond cognitive development, teachers need to be familiar with young children’s social, emotional, and motor development, all of which are relevant to mathematical development. To determine which puzzles and manipulative materials are helpful to support mathematical learning, for instance, teachers combine their knowledge of children’s cognition with the knowledge of fine7 motor development [45]. In deciding whether to let a 4-year-old struggle with a particular mathematical problem or to offer a clue, the teacher draws on more than an understanding of the cognitive demands involved. Important too are the teacher’s understanding of young children’s emotional development and her sensitivity to the individual child’s frustration tolerance and persistence [45, 46].

For some mathematical topics, researchers have identified a developmental continuum or learning path—a sequence indicating how particular concepts and skills build on others [44, 47, 48]. Snapshots taken from a few such sequences are given in the accompanying chart (pp. 19–21).

Research-based generalizations about what many children in a given grade or age range can do or understand are key in shaping curriculum and instruction, although they are only a starting point. Even with comparable learning opportunities, some children will grasp a concept earlier and others somewhat later. Expecting and planning for such individual variation are always important.

With the enormous variability in young children’s development, neither policymakers nor teachers should set a fixed timeline for children to reach each specific learning objective [49]. In addition to the risk of misclassifying individual children, highly specific timetables for skill acquisition pose another serious threat, especially when accountability pressures are intense. They tend to focus teachers’ attention on getting children to perform narrowly defined skills by a specified time, rather than on laying the conceptual groundwork that will serve children well in the long run. Such prescriptions often lead to superficial teaching and rote learning at the expense of real understanding. Under these conditions, children may develop only a shaky foundation for further mathematics learning [50–52].

4. Use curriculum and teaching practices that strengthen children’s problem-solving and reasoning processes as well as representing, communicating, and connecting mathematical ideas.

Problem solving and reasoning are the heart of mathematics. Teaching that promotes proficiency in these and other mathematical processes is consistent with national reports on mathematics education [5, 19, 53] and recommendations for early childhood practice [14, 46]. While content represents the what of early childhood mathematics education, the processes—problem solving, reasoning, communication, connections, and representation—make it possible for children to acquire content know edge [19]. These processes develop over time and when supported by well designed opportunities to learn.

Children's development and use of these processes are among the most longlasting and important achievements of mathematics education. Experiences and intuitive ideas become truly mathematical as the children reflect on them, represent them in various ways, and connect them to other ideas [19, 47].

The process of making connections deserves special attention. When children connect number to geometry (for example, by counting the sides of shapes, using arrays to understand number combinations, or measuring the length of their classroom), they strengthen concepts from both areas and build knowledge and beliefs about mathematics as a coherent system [19, 47]. Similarly, helping children connect mathematics to other subjects, such as science, develops knowledge of both subjects as well as knowledge of the wide applicability of mathematics. Finally and critically, teaching concepts and skills in a connected, integrated fashion tends to be particularly effective not only in the early childhood years [14, 23] but also in future learning [5, 54].

5. Ensure that the curriculum is coherent and compatible with known relationships and sequences of important mathematical ideas.

In developing early mathematics curriculum, teachers need to be alert to children's experiences, ideas, and creations [55, 56]. To create coherence and power in the curriculum, however, teachers also must stay focused on the "big ideas" of mathematics and on the connections and sequences among those ideas [23, 57].

The big ideas or vital understandings in early childhood mathematics are those that are mathematically central, accessible to children at their present level of understanding, and generative of future learning [28]. Research and expert practice indicate that certain concepts and skills are both challenging and accessible to young children [19]. National professional standards outline core ideas in each of five major content areas: number and operations, geometry, measurement, algebra (including patterns), and data analysis [19]. For example, the idea that the same pattern can describe different situations is a "big idea" within the content area of algebra and patterning.

These content areas and their related big ideas, however, are just a starting point. Where does one begin to build understanding of an idea such as "counting" or "symmetry," and where does one take this understanding over the early years of school? Articulating goals and standards for young children as a developmental or learning continuum is a particularly useful strategy in ensuring engagement with and mastery of important mathematical ideas [49]. In the key areas of mathematics, researchers have at least begun to map out trajectories or paths of learning—that is, the sequence in which young children develop mathematical understanding and skills [21, 58, 59]. The accompanying chart provides brief examples of learning paths in each content area and a few teaching strategies that promote children's progress along these paths. Information about such learning paths can support developmentally appropriate teaching, illuminating various avenues to understanding and guiding teachers in providing activities appropriate for children as individuals and as a group.

6. Provide for children's deep and sustained interaction with key mathematical ideas.

In many early childhood programs, mathematics makes only fleeting, random appearances. Other programs give mathematics adequate time in the curriculum but attempt to cover so many mathematical topics that the result is superficial and uninteresting to children. In a more effective third alternative, children encounter concepts in depth and in a logical sequence. Such depth and coherence allow children to develop, construct, test, and reflect on their mathematical understandings [10, 23, 59, 60]. This alternative also enhances teachers' opportunities to determine gaps in children's understanding and to take time to address these.

Because curriculum depth and coherence are important, unplanned experiences with mathematics are clearly not enough. Effective programs also include intentionally organized learning experiences that build children's understanding over time. Thus, early childhood educators need to plan for children's in-depth involvement with mathematical ideas, including helping families extend and develop these ideas outside of school.

Depth is best achieved when the program focuses on a number of key content areas rather than trying to cover every topic or skill with equal weight. As articulated in professional standards, researchers have identified number and operations, geometry, and measurement as areas particularly important for 3- to 6-year-olds [19]. These play an especially significant role in building the foundation for mathematics learning [47]. For this reason, researchers recommend that algebraic thinking and data analysis/probability receive somewhat less emphasis in the early years. The beginnings of ideas in these two areas, however, should be woven into the curriculum where they fit most naturally and seem most likely to promote understanding of the other topic areas [19]. Within this second tier of content areas, patterning (a component of algebra) merits special mention because it is accessible and interesting to young children, grows to undergird all algebraic thinking, and supports the development of number, spatial sense, and other conceptual areas.

7. Integrate mathematics with other activities and other activities with mathematics.

Young children do not perceive their world as if it were divided into separate cubbyholes such as "mathematics" or "literacy" [61]. Likewise, effective practice does not limit mathematics to one specified period or time of day. Rather, early childhood teachers help children develop mathematical knowledge throughout the day and across the curriculum. Children's everyday activities and routines can be used to introduce and develop important mathematical ideas [55, 59, 60, 62–67]. For example, when children are lining up, teachers can build in many opportunities to develop an understanding of mathematics. Children wearing something red can be asked to get in line first, those wearing blue to get in line second, and so on. Or children wearing both something red and sneakers can be asked to head up the line. Such opportunities to build important mathematical vocabulary and concepts abound in any classroom, and the alert teacher takes full advantage of them.

Also important is weaving mathematics into children's experiences with literature, language, science, social studies, art, movement, music, and all parts of the classroom environment. For example, there are books with mathematical concepts in the reading corner, and clipboards and wall charts are placed where children are engaged in science observation and recording (e.g., measuring and charting the weekly growth of plants) [65, 66, 68–71]. Projects also reach across subject-matter boundaries. Extended investigations offer children excellent opportunities to apply mathematics as well as to develop independence, persistence, and flexibility in making sense of real-life problems [19]. When children pursue a project or investigation, they encounter many mathematical problems and questions. With teacher guidance, children think about how to gather and record information and develop representations to help them in understanding and using the information and communicating their work to others [19, 72].

Another rationale for integrating mathematics throughout the day lies in easing competition for time in an increasingly crowded curriculum. Heightened attention to literacy is vital but can make it difficult for teachers to give mathematics and other areas their due. With a strong interdisciplinary curriculum, teachers can still focus on one area at times but also find ways to promote children's competence in literacy, mathematics, and other subjects within integrated learning experiences [73].

An important final note: As valuable as integration is within early childhood curriculum, it is not an end in itself. Teachers should ensure that the mathematics experiences woven throughout the curriculum follow logical sequences, allow depth and focus, and help children move forward in knowledge and skills. The curriculum should not become, in the name of integration, a grab bag of any mathematics-related experiences that seem to relate to a theme or project. Rather, concepts should be developed in a coherent, planful manner.

8. Provide ample time, materials, and teacher support for children to engage in play, a context in which they explore and manipulate mathematical ideas with keen interest.

Children become intensely engaged in play. Pursuing their own purposes, they tend to tackle problems that are challenging enough to be engrossing yet not totally beyond their capacities. Sticking with a problem—puzzling over it and approaching it in various ways—can lead to powerful learning. In addition, when several children grapple with the same problem, they often come up with different approaches, discuss, and learn from one another [74, 75]. These aspects of play tend to prompt and promote thinking and learning in mathematics and in other areas.

Play does not guarantee mathematical development, but it offers rich possibilities. Significant benefits are more likely when teachers follow up by engaging children in reflecting on and representing the mathematical ideas that have emerged in their play. Teachers enhance children's mathematics learning when they ask questions that provoke clarifications, extensions, and development of new understandings [19].

Block building offers one example of play's value for mathematical learning. As children build with blocks, they constantly accumulate experiences with the ways in which objects can be related, and these experiences become the foundation for a multitude of mathematical concepts—far beyond simply sorting and seriating. Classic unit blocks and other construction materials such as connecting blocks give children entry into a world where objects have predictable similarities and relationships [66, 76]. With these materials, children reproduce objects and structures from their daily lives and create abstract designs by manipulating pattern, symmetry, and other elements [77]. Children perceive geometric notions inherent in the blocks (such as two square blocks as the equivalent of one rectangular unit block) and the structures they build with them (such as symmetric buildings with parallel sides). Over time, children can be guided from an intuitive to a more explicit conceptual understanding of these ideas [66].

A similar progression from intuitive to explicit knowledge takes place in other kinds of play. Accordingly, early childhood programs should furnish materials and sustained periods of time that allow children to learn mathematics through playful activities that encourage counting, measuring, constructing with blocks, playing board and card games, and engaging in dramatic play, music, and art [19, 64].

Finally, the teacher can observe play to learn more about children's development and interests and use this knowledge to inform curriculum and instruction. With teacher guidance, an individual child's play interest can develop into a classroom-wide, extended investigation or project that includes rich mathematical learning [78–82]. In classrooms in which teachers are alert to all these possibilities, children's play continually stimulates and enriches mathematical explorations and learning.

9. Actively introduce mathematical concepts, methods, and language through a range of appropriate experiences and teaching strategies.

A central theme of this position statement is that early childhood curriculum needs to go beyond sporadic, hit-or-miss mathematics. In effective programs, teachers make judicious use of a variety of approaches, strategies, and materials to support children's interest and ability in mathematics.

Besides embedding significant mathematics learning in play, classroom routines, and learning experiences across the curriculum, an effective early mathematics program also provides carefully planned experiences that focus children's attention on a particular mathematical idea or set of related ideas. Helping children name such ideas as *horizontal* or *even and odd* as they find and create many examples of these categories provides children with a means to connect and refer to their just-emerging ideas [35, 37]. Such concepts can be introduced and explored in large- and small-group activities and learning centers. Small groups are particularly well suited to focusing children's attention on an idea. Moreover, in this setting the teacher is able to observe what each child does and does not understand and engage each child in the learning experience at his own level.

In planning for new investigations and activities, teachers should think of ways to engage children in revisiting concepts they have previously explored. Such experiences enable children to forge links between previously encountered mathematical ideas and new applications [19].

Even the way that teachers introduce and modify games can promote important mathematical concepts and provide opportunities for children to practice skills [55, 57]. For example, teachers can modify any simple board game in which players move along a path to make the game more mathematically powerful and more appropriate for children of differing developmental levels [55, 83].

Use of materials also requires intentional planning and involvement on the teacher's part. Computer technology is a good example [84]. Teachers need to intentionally select and use research-based computer tools that complement and expand what can be done with other media [59]. As with other instructional materials, choosing software and determining how best to incorporate computer use in the day-to-day curriculum requires thoughtful, informed decision-making in order for children's learning experiences to be rich and productive.

In short, mathematics is too important to be left to chance, and yet it must also be connected to children's lives. In making all of these choices, effective early childhood teachers build on children's informal mathematical knowledge and experiences, always taking children's cultural background and language into consideration [23].

10. Support children's learning by thoughtfully and continually assessing all children's mathematical knowledge, skills, and strategies.

Assessment is crucial to effective teaching [85]. Early childhood mathematics assessment is most useful when it aims to help young children by identifying their unique strengths and needs so as to inform teacher planning. Beginning with careful observation, assessment uses multiple sources of information gathered systematically over time—for example, a classroom book documenting the graphs made by children over several weeks. Mathematics assessment should follow widely accepted principles for varied and authentic early childhood assessment [85]. For instance, the teacher needs to use multiple assessment approaches to find out what each child understands—and may misunderstand. Child observation, documentation of children's talk, interviews, collections of children's work over time, and the use of open-ended questions and appropriate performance assessments to illuminate children's thinking are positive approaches to assessing mathematical strengths and needs [86, 87].

Careful assessment is especially important when planning for ethnically, culturally, and linguistically diverse young children and for children with special needs or disabilities. Effective teachers use information and insights gathered from assessment to plan and adapt teaching and curriculum. They recognize that even young children invent their own mathematical ideas and strategies and that children's ideas can be quite different from those of adults [44]. They interpret what the child is doing and thinking, and they attempt to see the situation from the child's point of view. With this basis in thoughtful assessment, teachers are able to make informed decisions about what the child might be able to learn from new experiences.

Reliance on a single group-administered test to document 3- to 6-year-old children's mathematical competence is counter to expert recommendations on assessment of young children [85, 88–91]. Educators must take care that assessment does not narrow the curriculum and inappropriately label children. If assessment results exclude some children from challenging learning activities, they undercut educational equity. Teachers and education policy makers need to stay in control of the assessment process, ensuring that it helps build mathematical competence and confidence. Well conceived, well implemented, continuous assessment is an indispensable tool in facilitating all children's engagement and success in mathematics.

Beyond the classroom

To support excellent early mathematics education, institutions, program developers, and policy makers should

1. Create more effective early childhood teacher preparation and continuing professional development.

Improving early childhood teacher preparation and ongoing professional development is an urgent priority. In mathematics, as in literacy and other areas, the challenges are formidable, but research-based solutions are available [14, 92–95]. To support children's mathematical proficiency, every early childhood teacher's professional preparation should include these connected components: (1) knowledge of the mathematical content and concepts most relevant for young children—including in-depth understanding of what children are learning now and how today's learning points toward the horizons of later learning [5]; (2) knowledge of young children's learning and development in all areas—including but not limited to cognitive development—and knowledge of the issues and topics that may engage children at different points in their development; (3) knowledge of effective ways of teaching mathematics to all young learners; (4) knowledge and skill in observing and documenting young children's mathematical activities and understanding; and (5) knowledge of resources and tools that promote mathematical competence and enjoyment [96].

Essential as this knowledge is, it can be brought to life only when teachers themselves have positive attitudes about mathematics. Lack of appropriate preparation may cause both preservice and experienced teachers to fail to see mathematics as a priority for young children and to lack confidence in their ability to teach mathematics effectively [97]. Thus, both preservice education and continuing professional development experiences need to place greater emphasis on encouraging teachers' own enjoyment and confidence, building positive mathematical attitudes and dispositions.

Even graduates of four-year early childhood programs with state licensure usually lack adequate preparation in mathematics. State legislatures often address their concern over teachers' weak background in mathematics by simply increasing the number of required mathematics courses needed for teacher licensure.

This remedy lacks research support [5, 92]. Credit hours or yearly training requirements do little or nothing unless the content and delivery of professional development are designed to produce desired outcomes for teachers and children [93].

Teachers of young children should learn the mathematics content that is directly relevant to their professional role. But content alone is not enough. Effective professional programs weave together mathematics content, pedagogy, and knowledge of child development and family relationships [98]. When high-quality, well supervised field work is integrated throughout a training program, early childhood teachers can apply their knowledge in realistic contexts. Courses or inservice training should be designed to help teachers develop a deep understanding of the mathematics they will teach and the habits of mind of a mathematical thinker. Courses, practicum experiences, and other training should strengthen teachers' ability to ask young children the kinds of questions that stimulate mathematical thinking. Effective professional development, whether preservice or inservice, should also model the kind of flexible, interactive teaching styles that work well with children [92].

Preservice and inservice professional development present somewhat differing challenges. In preservice education, the major challenge is to build a sound, well integrated knowledge base about mathematics, young children's development and learning, and classroom practices [5]. Inservice training shares this challenge but also carries risks of superficiality and fragmentation.

To avoid these risks, inservice professional development needs to move beyond the one-time workshop to deeper exploration of key mathematical topics as they connect with young children's thinking and with classroom practices. Inservice professional development in mathematics appears to have the greatest impact on teacher learning if it incorporates six features: teacher networking or study groups; sustained, intensive programs; collective participation of staff who work in similar settings; content focused both on what and how to teach; active learning techniques; and professional development as part of a coherent program of teacher learning [5, 99]. Innovative and effective professional development models may use a variety of research-based approaches. In addition, classroom-based inquiry, team teaching by mathematics and early childhood education specialists, discussion of case studies, and analysis of young children's work samples tend to strengthen teachers' confidence and engagement in early childhood mathematics [5, 97, 99, 100].

Delivering this kind of ongoing professional development requires a variety of innovative strategies. For early childhood staff living in isolated communities or lacking knowledgeable trainers, distance learning with local facilitators is a promising option. Literacy initiatives are increasingly using itinerant or school-wide specialists; similarly, mathematics education specialists could offer resources to a number of early childhood programs. Partnerships between higher education institutions and local early childhood programs can help provide this support. Finally, school-district-sponsored professional development activities that include participants from community child care centers, family child care, and Head Start programs along with public school kindergarten/primary teachers would build coherence and continuity for teachers and for children's mathematical experiences.

2. Use collaborative processes to develop well aligned systems of appropriate high-quality standards, curriculum, and assessment.

In mathematics, as in other domains, the task of developing curriculum and related goals and assessments has become the responsibility not only of the classroom teacher but also of other educators and policy makers. State agencies, school districts, and professional organizations are engaged in standards setting, defining desired educational and developmental outcomes for children below kindergarten age [13]. This trend represents an opportunity to improve early childhood mathematics education but also presents a challenge. The opportunity is to develop a coherent, developmentally appropriate, and well aligned system that offers teachers a framework to guide their work. The challenge, especially at the preschool and kindergarten levels, is to ensure that such a framework does not stifle innovation, put children into inappropriate categories, ignore important individual or cultural differences, or result in narrowed and superficial teaching that fails to give children a solid foundation of understanding [49].

To avoid these risks, state agencies and others must work together to develop more coherent systems of standards, curriculum, instruction, and assessment that support the development of mathematical proficiency. To build coherence between preschool and early elementary mathematics, the processes of setting standards and developing early childhood curriculum and assessment systems must include the full range of stakeholders. Participants should include not only public school teachers and administrators but also personnel from center-based programs and family child care, private and public prekindergarten, and Head Start, as well as others who serve young children and their families. Families too should participate as respected partners. Relevant expertise should be sought from professional associations and other knowledgeable sources.

As in all effective standards-setting efforts, early childhood mathematics standards should be coupled with an emphasis on children's opportunities to learn, not just on expectations for their performance. Standards also should be accompanied by descriptions of what young children might be expected to accomplish along a flexible developmental continuum [49]. Standards for early childhood mathematics should connect meaningfully but not rigidly with curriculum. Assessment also should align with curriculum and with standards, following the principles articulated by national groups concerned with appropriate assessment for young children [88–91].

District- or program-level educators are often responsible for selecting or developing curriculum. Decision makers can be guided by the general criteria for curriculum adoption articulated in the position statement jointly adopted by NAEYC and the National Association of Early Childhood Specialists in State Departments of Education [85]. In addition, decision makers should insist that any mathematics curriculum considered for adoption has been extensively field tested and evaluated with young children.

3. Design institutional structures and policies that support teachers' ongoing learning, teamwork, and planning.

National reports stress the need for teacher planning and collaboration [5, 7, 101, 102], yet few early childhood programs have the structures and supports to enable these processes to take place regularly. Teachers of young children face particular challenges in planning mathematics activities. Early childhood teachers work in diverse settings, and some of these settings pose additional obstacles to teamwork and collaboration. Many early childhood programs, in or out of public school settings, have little or no time available for teacher planning, either individually or in groups. Team meetings and staff development activities occur infrequently.

The institutional divide between teachers in child care, Head Start, or preschool programs and those in public kindergarten and primary programs presents a barrier to the communication necessary for a coherent mathematics curriculum. Without communication opportunities, preschool teachers often do not know what kindergarten programs expect, and early elementary teachers may have little idea of the content or pedagogy used in prekindergarten mathematics education. New strategies and structures, such as joint inservice programs and classroom visits, could support these linkages.

In addition, many programs have limited access to specialists who might help teachers as they try to adopt new approaches to early childhood mathematics. Administrators need to reexamine their allocation of resources and their scheduling practices, keeping in mind the value of investing in teacher planning time.

4. Provide the resources necessary to overcome the barriers to young children's mathematical proficiency at the classroom, community, institutional, and system-wide levels.

A variety of resources, some financial and some less tangible, are needed to support implementation of this position statement's recommendations. Partnerships among the business, philanthropic, and government sectors at the national, state, and local levels will improve teaching and learning in all communities, including those that lack equitable access to mathematics education. Universally available early childhood mathematics education can occur only in the context of a comprehensive, well financed system of high-quality early education, including child care, Head Start, and prekindergarten programs [103–106]. To support universal mathematical proficiency, access to developmentally and educationally effective programs of early education, supported by adequate resources, should be available to all children.

Improvement of early childhood mathematics education also requires substantial investment in teachers' professional development. The mathematics knowledge gap must be bridged with the best tools, including resources for disseminating models of effective practice, videos showing excellent mathematics pedagogy in real-life settings, computer-based professional development resources, and other materials. In addition, resources are needed to support teachers' involvement in professional conferences, college courses, summer institutes, and visits to model sites.

To support effective teaching and learning, mathematics-rich classrooms require a wide array of materials for young children to explore and manipulate [45, 59, 107]. Equity requires that all programs, not just those serving affluent communities, have these resources.

Finally, resources are needed to support families as partners in developing their young children's mathematical proficiency. The growing national awareness of families' central role in literacy development is a good starting point from which to build awareness of families' equally important role in mathematical development [108, 109]. Public awareness campaigns, distribution of materials in ways similar to the successful Reach Out and Read initiative, computer-linked as well as school-based meetings for families, Family Math Nights, and take-home activities such as mathematics games and manipulative materials tailored to the ages, interests, languages, and cultures of the children—these are only a few examples of the many ways in which resources can support families' engagement in their young children's mathematical learning [110, see also the online "Family Math" materials at www.lhs.berkeley.edu/equals/FMnetwork.htm and other resources at www.nctm.org/corners/family/index.htm].

Conclusion

A positive attitude toward mathematics and a strong foundation for mathematics learning begin in early childhood. These good beginnings reflect all the characteristics of good early childhood education: deep understanding of children's development and learning; a strong community of teachers, families, and children; research-based knowledge of early childhood curriculum and teaching practices; continuous assessment in the service of children's learning; and an abiding respect for young children's families, cultures, and communities.

To realize this vision, educators, administrators, policy makers, and families must work together—raising awareness of the importance of mathematics in early education, informing others about sound approaches to mathematical teaching and learning, and developing essential resources to support high-quality, equitable mathematical experiences for all young children.

References

1. Haycock, K., & S. Huang. 2001. Are today's high school graduates ready? *Thinking K–16* 5 (1): 3–17.
2. Haycock, K. 2001. Youth at the crossroads: Facing high school and beyond. *Thinking K–16* 5 (1): 1–2.
3. Schoenfeld, A.H. 2002. Making mathematics work for all children: Issues of standards, testing, and equity. *Educational Researcher* 31: 13–25.
4. The Education Trust. 2001. Actions for communities and states. *Thinking K–16* 5 (1): 18–21.
5. Kilpatrick, J., J. Swafford, & B. Findell. 2001. *Adding it up: Helping children learn mathematics.* Washington, DC: National Academy Press.
6. U.S. Department of Labor Bureau of Labor Statistics. 2000. The outlook for college graduates, 1998–2008. *In Getting ready pays off!,* U.S. DOE, October 2000, & *BLS, Occupational Employment Projections to 2008,* in NAB, *Workforce Economics* 6 (1).
7. Glenn Commission. 2000. *Before it's too late: A report to the nation from the National Commission on Mathematics and Science Teaching for the 21st Century.* Washington, DC: U.S. Department of Education.
8. Mullis, I.V.S., M.O. Martin, A.E. Beaton, E.J. Gonzalez, D.L. Kelly, & T.A. Smith. 1997. *Mathematics achievement in the primary school years: IEA's Third International Mathematics and Science Study (TIMSS).* Chestnut Hill, MA: Center for the Study of Testing, Evaluation, and Educational Policy, Boston College.
9. Mullis, I.V.S., M.O. Martin, E.J. Gonzalez, K.D. Gregory, R.A. Garden, K.M. O'Connor, S.J. Chrostowski, & T.A. Smith. 2000. *TIMSS 1999 international mathematics report.* Boston: International Study Center, Boston College, Lynch School of Education.
10. Fuson, K.C., W.M. Carroll, & J.V. Drueck. 2000. Achievement results for second and third graders using the standards-based curriculum Everyday Mathematics. *Journal for Research in Mathematics Education* 31: 277–95.
11. Mullis, I.V.S., M.O. Martin, E.J. Gonzalez, K.M. O'Connor, S.J. Chrostowski, K.D. Gregory, R.A. Garden, & T.A. Smith. 2001. *Mathematics benchmarking report: TIMSS 1999—Eighth grade.* Chestnut Hill, MA: International Association for the Evaluation of Educational Achievement.
12. Riordan, J.E., & P.E. Noyce. 2001. The impact of two standards-based mathematics curricula on student achievement in Massachusetts. *Journal for Research in Mathematics Education* 32: 368–98.
13. Education Week. 2002. Quality Counts 2002: Building blocks for success: State efforts in early-childhood education. *Education Week* (Special issue) 21 (17).
14. Bowman, B.T., M.S. Donovan, & M.S. Burns, eds. 2001. *Eager to learn: Educating our preschoolers.* Washington, DC: National Academy Press.
15. Denton, K., & J. West. 2002. *Children's reading and mathematics achievement in kindergarten and first grade.* Washington, DC: National Center for Education Statistics.
16. Natriello, G., E.L. McDill, & A.M. Pallas. 1990. *Schooling disadvantaged children: Racing against catastrophe.* New York: Teachers College Press.
17. Starkey, P., & A. Klein. 1992. Economic and cultural influence on early mathematical development. In *New directions in child and family research: Shaping Head Start in the nineties,* eds. F. Lamb-Parker, R. Robinson, S. Sambrano, C. Piotrkowski, J. Hagen, S. Randolph, & A. Baker, 440. Washington, DC: Administration on Children, Youth and Families (DHHS).
18. Shonkoff, J.P., & D.A. Phillips, eds. 2000. *From neurons to neighborhoods: The science of early childhood development.* Washington, DC: National Academy Press.
19. National Council of Teachers of Mathematics. 2000. *Principles and standards for school mathematics.* Reston, VA: Author.
20. Seo, K.-H., & H.P. Ginsburg. 2004. What is developmentally appropriate in early childhood mathematics education? In *Engaging young children in mathematics: Standards for early childhood mathematics education,* eds. D.H. Clements, J. Sarama, & A.-M. DiBiase, 91–104. Mahwah, NJ: Lawrence Erlbaum.
21. Baroody, A.J. 2004. The role of psychological research in the development of early childhood mathematics standards. In *Engaging young children in mathematics: Standards for early childhood mathematics education,* eds. D.H. Clements, J. Sarama, & A.-M. DiBiase, 149–72. Mahwah, NJ: Lawrence Erlbaum.
22. Clements, D.H., S. Swaminathan, M.-A. Hannibal, & J. Sarama. 1999. Young children's concepts of shape. *Journal for Research in Mathematics Education* 30: 192–212.
23. Fuson, K.C. 2004. Pre-K to grade 2 goals and standards: Achieving 21st century mastery for all. In *Engaging young children in mathematics: Standards for early childhood mathematics education,* eds. D.H. Clements, J. Sarama, & A.-M. DiBiase, 105–48. Mahwah, NJ: Lawrence Erlbaum.
24. Gelman, R. 1994. *Constructivism and supporting environments. In Implicit and explicit knowledge: An educational approach,* ed. D. Tirosh, 55–82. Norwood, NJ: Ablex.

25. Ginsburg, H.P., A. Klein, & P. Starkey. 1998. The development of children's mathematical thinking: Connecting research with practice. In *Handbook of child psychology, Volume 4: Child psychology in practice,* eds. W. Damon, I.E. Sigel, & K.A. Renninger, 401–76. New York: John Wiley & Sons.
26. Piaget, J., and B. Inhelder. 1967. *The child's conception of space.* New York: W.W. Norton.
27. Steffe, L.P. 2004. PSSM from a constructivist perspective. In *Engaging young children in mathematics: Standards for early childhood mathematics education,* eds. D.H. Clements, J. Sarama, & A.-M. DiBiase, 221–52. Mahwah, NJ: Lawrence Erlbaum.
28. Clements, D.H., & Conference Working Group. 2004. Part one: Major themes and recommendations. In *Engaging young children in mathematics: Standards for early childhood mathematics education,* eds. D.H. Clements, J. Sarama, & A.-M. DiBiase, 7–76. Mahwah, NJ: Lawrence Erlbaum.
29. Copple, C.E. 2004. Math curriculum in the early childhood context. In *Engaging young children in mathematics: Standards for early childhood mathematics education,* eds. D.H. Clements, J. Sarama, & A.-M. DiBiase, 83–90. Mahwah, NJ: Lawrence Erlbaum.
30. Geary, D.C. 1994. *Children's mathematical development: Research and practical applications. Washington,* DC: American Psychological Association.
31. Ginsburg, H.P., S. Pappas, & K.-H. Seo. 2001. Everyday mathematical knowledge: Asking young children what is developmentally appropriate. In *Psychological perspectives on early childhood education: Reframing dilemmas in research and practice,* ed. S.L. Golbeck, 181–219. Mahwah, NJ: Lawrence Erlbaum.
32. Han, Y., & H.P. Ginsburg. 2001. Chinese and English mathematics language: The relation between linguistic clarity and mathematics performance. *Mathematical Thinking and Learning* 3: 201–20.
33. Miura, I.T., C.C. Kim, C.-M. Chang, & Y. Okamoto. 1988. Effects of language characteristics on children's cognitive representation of number: Cross-national comparisons. *Child Development* 59: 1445–50.
34. Park, M. 2000. Linguistic influence on numerical development. *The Mathematics Educator* 10 (1): 19–24.
35. Berk, L.E., & A. Winsler. 1995. *Scaffolding children's learning: Vygotsky and early childhood education.* Washington, DC: NAEYC.
36. Heath, S.B. 1983. *Ways with words: Language, life, and work in communities and classrooms.* Cambridge, UK: Cambridge University Press.
37. Vygotsky, L.S. [1934] 1986. *Thought and language.* Cambridge, MA: MIT Press.
38. Razel, M., & B.-S. Eylon. 1990. Development of visual cognition: Transfer effects of the Agam program. *Journal of Applied Developmental Psychology* 11: 459–85.
39. Bredekamp, S., & T. Rosegrant. 1995. *Reaching potentials: Transforming early childhood curriculum and assessment. Volume 2.* Washington, DC: NAEYC.
40. Clements, D.H. 1999. *Geometric and spatial thinking in young children. In Mathematics in the early years,* ed. J.V. Copley, 66–79. Reston, VA: National Council of Teachers of Mathematics.
41. Starkey, P., & R.G. Cooper Jr. 1980. Perception of numbers by human infants. *Science* 210: 1033–35.
42. Starkey, P., E.S. Spelke, & R. Gelman. 1990. Numerical abstraction by human infants. *Cognition* 36: 97–128.
43. Trafton, P.R., & A. Andrews. 2002. *Little kids—Powerful problem solvers: Math stories from a kindergarten classroom.* Portsmouth, NH: Heinemann.
44. Steffe, L.P., & P. Cobb. 1988. *Construction of arithmetical meanings and strategies.* New York: Springer-Verlag.
45. Bronson, M.B. 1995. *The right stuff for children birth to 8: Selecting play materials to support development.* Washington, DC: NAEYC.
46. Copple, C., & S. Bredekamp. 2009. *Developmentally appropriate practice in early childhood programs serving children birth through age 8*. 3d ed. Washington, DC: NAEYC.
47. Clements, D.H., J. Sarama, & A.-M. DiBase, eds. 2004. *Engaging young children in mathematics: Standards for early childhood mathematics education.* Mahwah, NJ: Lawrence Erlbaum.
48. Gravemeijer, K.P.E. 1999. How emergent models may foster the constitution of formal mathematics. *Mathematical Thinking and Learning* 1: 155–77.
49. Bredekamp, S. 2004. Standards for preschool and kindergarten mathematics education. In *Engaging young children in mathematics: Standards for early childhood mathematics education,* eds. D.H. Clements, J. Sarama, & A.-M. DiBiase, 77–82. Mahwah, NJ: Lawrence Erlbaum.
50. Carpenter, T.P., M.L. Franke, V. Jacobs, & E. Fennema. 1998. A longitudinal study of invention and understanding in children's multidigit addition and subtraction. *Journal for Research in Mathematics Education* 29: 3–20.
51. Erlwanger, S.H. 1973. Benny's conception of rules and answers in IPI mathematics. *Journal of Children's Mathematical Behavior* 1 (2): 7–26.
52. Kamii, C.K., & A. Dominick. 1998. The harmful effects of algorithms in grades 1–4. In *The teaching and learning of algorithms in school mathematics,* eds. L.J. Morrow & M.J. Kenney, 130–40. Reston, VA: National Council of Teachers of Mathematics.
53. National Research Council. 1989. *Everybody counts: A report to the nation on the future of mathematics education.* Washington, DC: National Academy Press.

54. Sophian, C. 2004. A prospective developmental perspective on early mathematics instruction. In *Engaging young children in mathematics: Standards for early childhood mathematics education,* eds. D.H. Clements, J. Sarama, & A.-M. DiBiase, 253–66. Mahwah, NJ: Lawrence Erlbaum.
55. Kamii, C.K., & L.B. Housman. 1999. *Young children reinvent arithmetic: Implications of Piaget's theory.* 2d ed. New York: Teachers College Press.
56. Steffe, L.P. 1990. Mathematics curriculum design: A constructivist's perspective. In *Transforming children's mathematics education: International perspectives*, eds. L.P. Steffe & T. Wood, 389–98. Hillsdale, NJ: Lawrence Erlbaum.
57. Griffin, S., R. Case, & A. Capodilupo. 1995. Teaching for understanding: The importance of the central conceptual structures in the elementary mathematics curriculum. In *Teaching for transfer: Fostering generalization in learning,* eds. A. McKeough, J. Lupart, & A. Marini. Mahwah, NJ: Lawrence Erlbaum.
58. Clements, D.H. 2004. Linking research and curriculum development. In *Handbook of international research in mathematics education,* ed. L.D. English. Mahwah, NJ: Lawrence Erlbaum.
59. Sarama, J. 2004. Technology in early childhood mathematics: Building Blocks™ as an innovative technology-based curriculum. In *Engaging young children in mathematics: Standards for early childhood mathematics education,* eds. D.H. Clements, J. Sarama, & A.-M. DiBiase, 361–76. Mahwah, NJ: Lawrence Erlbaum.
60. Griffin, S. 2004. Number Worlds: A research-based mathematics program for young children. In *Engaging young children in mathematics: Standards for early childhood mathematics education,* eds. D.H. Clements, J. Sarama, & A.-M. DiBiase, 325–42. Mahwah, NJ: Lawrence Erlbaum.
61. Clements, D.H. 2001. Mathematics in the preschool. *Teaching Children Mathematics* 7: 270–75.
62. Basile, C.G. 1999. The outdoors as a context for mathematics in the early years. In *Mathematics in the early years,* ed. J.V. Copley, 156–61. Reston, VA: National Council of Teachers of Mathematics.
63. Casey, M.B., R.L. Nuttall, & E. Pezaris. 1999. Evidence in support of a model that predicts how biological and environmental factors interact to influence spatial skills. *Developmental Psychology* 35 (5): 1237–47.
64. Hildebrandt, C., & B. Zan. 2002. Using group games to teach mathematics. In *Developing constructivist early childhood curriculum: Practical principles and activities,* ed. R. DeVries, 193–208. New York: Teachers College Press.
65. Hong, H. 1999. Using storybooks to help young children make sense of mathematics. In *Mathematics in the early years,* ed. J.V. Copley, 162–68. Reston, VA: National Council of Teachers of Mathematics.
66. Leeb-Lundberg, K. 1996. The block builder mathematician. In *The block book,* ed. E.S. Hirsh. Washington, DC: NAEYC.
67. Shane, R. 1999. Making connections: A "number curriculum" for preschoolers. In *Mathematics in the early years,* ed. J.V. Copley, 129–34. Reston, VA: National Council of Teachers of Mathematics.
68. Coates, G.D., & J. Franco. 1999. Movement, mathematics, and learning: Experiences using a family learning model. In *Mathematics in the early years,* ed. J.V. Copley, 169–74. Reston, VA: National Council of Teachers of Mathematics.
69. Copley, J.V. 2010. *The young child and mathematics.* 2d ed. Washington, DC: NAEYC.
70. Goodway, J.D., M. E. Rudisill, M.L. Hamilton, & M.A. Hart. 1999. Math in motion. In *Mathematics in the early years,* ed. J.V. Copley, 175–81. Reston, VA: National Council of Teachers of Mathematics.
71. Kim, S.L. 1999. Teaching mathematics through musical activities. In *Mathematics in the early years,* ed. J.V. Copley, 146–50. Reston, VA: National Council of Teachers of Mathematics.
72. Helm, J.H., S. Beneke, & K. Steinheimer. 1998. *Windows on learning: Documenting young children's work.* New York: Teachers College Press.
73. Balfanz, R. 2001. *Developing and assessing young children's mathematical knowledge.* Washington, DC: National Institute for Early Childhood Professional Development & NAEYC.
74. Nastasi, B.K., & D.H. Clements. 1991. Research on cooperative learning: Implications for practice. *School Psychology Review* 20: 110–31.
75. Yackel, E., P. Cobb, & T. Wood. 1991. Small group interactions as a source of learning opportunities in second grade mathematics. *Journal for Research in Mathematics Education* 22: 390–408.
76. Pratt, C. 1948. *I learn from children.* New York: Simon and Schuster.
77. Ginsburg, H.P., N. Inoue, & K.-H. Seo. 1999. Young children doing mathematics: Observations of everyday activities. In *Mathematics in the early years,* ed. J.V. Copley, 88–100. Reston, VA: National Council of Teachers of Mathematics.
78. Edwards, C., L. Gandini, & G. Forman. 1993. *The hundred languages of children: The Reggio Emilia approach to early childhood education.* Norwood, NJ: Ablex.
79. Helm, J.H., & L.G. Katz. 2001. Y*oung investigators: The project approach in the early years.* 2d ed. New York: Teachers College Press.

80. Jones, E., & J. Nimmo. 1994. *Emergent curriculum.* Washington, DC: NAEYC.

81. Katz, L.G., & S.C. Chard, 2000. *Engaging children's minds: The project approach. 2d ed.* Stamford, CT: Ablex.

82. Malaguzzi, L. 1997. *Shoe and meter.* Reggio Emilia, Italy: Reggio Children.

83. Charlesworth, R. 2000. *Experiences in math for young children.* Albany, NY: Delmar.

84. Clements, D.H. 1999. Young children and technology. In *Dialogue on early childhood science, mathematics, and technology education,* ed. G.D. Nelson, 92–105, Washington, DC: American Association for the Advancement of Science.

85. NAEYC & the National Association of Early Childhood Specialists in State Departments of Education. 1991. Guidelines for appropriate curriculum content and assessment in programs serving children ages 3 through 8. *Young Children* 46 (3): 21–38.

86. Chittenden, E. 1991. Authentic assessment, evaluation, and documentation of student performance. In *Expanding student assessment,* ed. V. Perrone, 22–31. Alexandria, VA: Association for Supervision and Curriculum Development.

87. Lindquist, M.M., & J.N. Joyner. 2004. Moving ahead in support of young children's mathematical learning: Recommendations to conference organizers and participants. In *Engaging young children in mathematics: Standards for early childhood mathematics education,* eds. D.H. Clements, J. Sarama, & A.-M. DiBiase, 449–56. Mahwah, NJ: Lawrence Erlbaum.

88. Horm-Wingerd, D.M., P.C. Winter, & P. Plofchan. 2000. *Primary level assessment for IASA Title I: A call for discussion.* Washington, DC: Council of Chief State School Officers.

89. National Association of School Psychologists. 1999. *Position statement on early childhood assessment.* Washington, DC: Author.

90. National Education Goals Panel. 1998. *Principles and recommendations for early childhood assessments* (submitted to NEGP by the Goal 1 Early Childhood Assessments Resource Group, eds. L. Shepard, S.L. Kagan, & E. Wurtz). Washington, DC: U.S. Government Printing Office.

91. Neisworth, J.T., & S.J. Bagnato. 2001. Recommended practices in assessment. In *DEC recommended practices in early intervention/early childhood special education,* eds. S. Sandall, M.E. McLean, & B.J. Smith, 17–28. Longmont, CO: Sopris West.

92. Conference Board of the Mathematical Sciences. 2001. *The mathematical education of teachers, part one.* Providence, RI: Mathematical Association of America.

93. NAEYC. 2001. *NAEYC standards for early childhood professional preparation.* Washington, DC: Author.

94. Peisner-Feinberg, E.S., R. Clifford, M. Culkin, C. Howes, & S.L. Kagan. 1999. *The children of the Cost, Quality, and Outcomes Study go to school.* Chapel Hill, NC: Frank Porter Graham Child Development Center, University of North Carolina at Chapel Hill.

95. U.S. Department of Education. 1999. *New teachers for a new century: The future of early childhood professional preparation.* Washington, DC: Author.

96. Copley, J.V., & Y. Padròn. 1999. Preparing teachers of young learners: Professional development of early childhood teachers in mathematics and science. In *Dialogue on early childhood science, mathematics, and technology education,* ed. G.D. Nelson, 117–29. Washington, DC: American Association for the Advancement of Science.

97. Sarama, J., & A.-M. DiBiase. 2004. The professional development challenge in preschool mathematics. In *Engaging young children in mathematics: Standards for early childhood mathematics education,* eds. D.H. Clements, J. Sarama, & A.-M.DiBiase, 415–48. Mahwah, NJ: Lawrence Erlbaum.

98. Baroody, A.J., & R.T. Coslick. 1998. *Fostering children's mathematical power: An investigative approach to K–8 mathematics instruction.* Mahwah, NJ: Lawrence Erlbaum.

99. Copley, J.V. 2004. The early childhood collaborative: A professional development model to communicate and implement the standards. In *Engaging young children in mathematics: Standards for early childhood mathematics education,* eds. D.H. Clements, J. Sarama, & A.-M. DiBiase, 401–14. Mahwah, NJ: Lawrence Erlbaum.

100. Ball, D., & D. Cohen. 1999. Developing practice, developing practitioners: Toward a practice-based theory of professional education. In *Teaching as the learning profession,* eds. L. Darling-Hammond & G. Sykes. San Francisco: Jossey-Bass.

101. Bransford, J.D., A.L. Brown, & R.R. Cocking, eds. 1999. *How people learn.* Washington, DC: National Academy Press.

102. Darling-Hammond, L. 1990. Instructional policy into practice: "The power of the bottom over the top." *Educational Evaluation and Policy Analysis* 12 (3): 339–47.

103. Barnett, W.S., & L. Masse. 2001. *Financing early care and education in the United States: CEER policy brief.* New Brunswick, NJ: Center for Early Education Research.

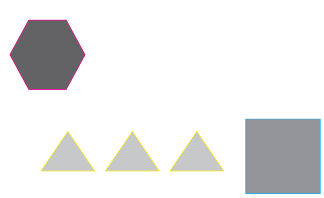
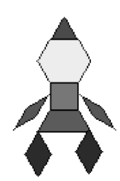
104. Brandon, R.N., S.L. Kagan, & J.M. Joesch. 2000. *Design choices: Universal financing for early care and education.* Seattle: University of Washington.

105. Mitchell, A., L. Stoney, & H. Dichter. 2001. *Financing child care in the United States: An expanded catalog of current strategies.* 2d ed. Kansas City, MO: Ewing Marion Kauffman Foundation.
106. Office of Economic Cooperation and Development. 2000. *OECD country note: Early childhood education and care policy in the United States of America.* Washington, DC: Office for Educational Research and Improvement.
107. Clements, D.H. 2003. Teaching and learning geometry. In *A research companion to Principles and Standards for School Mathematics,* eds. J. Kilpatrick, W.G. Martin, & D.E. Schifter. Reston, VA: National Council of Teachers of Mathematics.
108. Moll, L.C., C. Armanti, D. Neff, & N. Gonzalez. 1992. Funds of knowledge for teaching: Using a qualitative approach to connect homes and classrooms. *Theory into Practice* 31: 132–41.
109. Starkey, P., & A. Klein. 2000. Fostering parental support for children's mathematical development: An intervention with Head Start families. *Early Education and Development* 11: 659–80.
110. Edge, D. 2000. *Involving families in school mathematics: Readings from Teaching Children Mathematics, Mathematics Teaching in the Middle School, and Arithmetic Teacher*. Reston, VA: National Council of Teachers of Mathematics.

Learning PATHS and Teaching STRATEGIES in Early Mathematics

The research base for sketching a picture of children's mathematical development varies considerably from one area of mathematics to another. Outlining a learning path, moreover, does not mean we can predict with confidence where a child of a given age will be in that sequence. Developmental variation is the norm, not the exception. However, children do tend to follow similar *sequences*, or *learning paths*, as they develop. This chart illustrates in each area some things that *many* children know and do—early and late in the 3–6 age range. These are, then, simply two points along the learning path that may have many steps in between. For each content area, the Sample Teaching Strategies column shows a few of the many teacher actions that promote learning when used within a classroom context that reflects the recommendations set forth in this NAEYC/NCTM position statement. In general, they are helpful strategies, with minor adaptations, across the age range.

Content Area	**Examples of Typical Knowledge and Skills** From Age 3 →	Age 6	**Sample Teaching Strategies**
Number and operations	Counts a collection of one to four items and begins to understand that the last counting word tells *how many.*	Counts and produces (counts out) collections up to 100 using groups of 10.	Models counting of small collections and guides children's counting in everyday situations, emphasizing that we use one counting word for each object: ♡ ♡ ♡ "One . . . two . . . three . . ." Models counting by 10s while making groups of 10s (e.g., 10, 20, 30 . . . or 14, 24, 34 . . .).
	Quickly "sees" and labels collections of one to three with a number.	Quickly "sees" and labels with the correct number "patterned" collections (e.g., dominoes) and unpatterned collections of up to about six items.	Gives children a brief glimpse (a couple of seconds) of a small collection of items and asks how many there are.

Content Area	Examples of Typical Knowledge and Skills From Age 3 →	Age 6	Sample Teaching Strategies
Number and operations	Adds and subtracts nonverbally when numbers are very low. For example, when one ball and then another are put into the box, expects the box to contain two balls.	Adds or subtracts using counting-based strategies such as counting on (e.g., adding 3 to 5, says "Five . . . , six, seven, eight"), when numbers and totals do not go beyond 10.	Tells real-life stories involving numbers and a problem. Asks *how many* questions (e.g., "How many are left?" "How many are there now?" "How many did they start with?" "How many were added?"). Shows children the use of objects, fingers, counting on, guessing, and checking to solve problems.
Geometry and spatial sense	Begins to match and name 2-D and 3-D shapes, first only with same size and orientation, then shapes that differ in size and orientation (e.g., a large triangle sitting on its point versus a small one sitting on its side).	Recognizes and names a variety of 2-D and 3-D shapes (e.g., quadrilaterals, trapezoids, rhombi, hexagons, spheres, cubes) in any orientation. Describes basic features of shapes (e.g., number of sides or angles).	Introduces and labels a wide variety of shapes (e.g., skinny triangles, fat rectangles, prisms) that are in a variety of positions (e.g., a square or a triangle standing on a corner, a cylinder "standing up" or horizontal). Involves children in constructing shapes and talking about their features.
	Uses shapes, separately, to create a picture.  Describes object locations with spatial words such as *under* and *behind* and builds simple but meaningful "maps" with toys such as houses, cars, and trees.	Makes a picture by combining shapes.  Builds, draws, or follows simple maps of familiar places, such as the classroom or playground.	Encourages children to make pictures or models of familiar objects using shape blocks, paper shapes, or other materials. Encourages children to make and talk about models with blocks and toys. Challenges children to mark a path from a table to the wastebasket with masking tape, then draw a map of the path, adding pictures of objects appearing along the path, such as a table or easel.

Content Area	Examples of Typical Knowledge and Skills From Age 3 ⟶ Age 6		Sample Teaching Strategies
Measurement	Recognizes and labels measurable attributes of objects (e.g., "I need a long string," "Is this heavy?"). Begins to compare and sort according to these attributes (e.g., *more/less*, *heavy/light*; "This block is too short to be the bridge").	Tries out various processes and units for measurement and begins to notice different results of one method or another (e.g., what happens when we *don't* use a standard unit). Makes use of nonstandard measuring tools or uses conventional tools such as a cup or ruler in nonstandard ways (e.g., "It's three rulers long").	Uses comparing words to model and discuss measuring (e.g. "This book feels heavier than that block," "I wonder if this block tower is taller than the desk?"). Uses and creates situations that draw children's attention to the problem of measuring something with two different units (e.g., making garden rows "four shoes" apart, first using a teacher's shoe and then a child's shoe).
Pattern/algebraic thinking	Notices and copies simple repeating patterns, such as a wall of blocks with long, short, long, short, long, short, long. . . .	Notices and discusses patterns in arithmetic (e.g., adding one to any number results in the next "counting number").	Encourages, models, and discusses patterns (e.g., "What's missing?" "Why do you think that is a pattern?" "I need a blue next"). Engages children in finding color and shape patterns in the environment, number patterns on calendars and charts (e.g., with the numerals 1–100), patterns in arithmetic (e.g., recognizing that when zero is added to a number, the sum is always that number).
Displaying and analyzing data	Sorts objects and counts and compares the groups formed. Helps to make simple graphs (e.g., a pictograph formed as each child places her own photo in the row indicating her preferred treat—pretzels or crackers).	Organizes and displays data through simple numerical representations such as bar graphs and counts the number in each group.	Invites children to sort and organize collected materials by color, size, shape, etc. Asks them to compare groups to find which group has the most. Uses "not" language to help children analyze their data (e.g., "All of these things are red, and these things are NOT red"). Works with children to make simple numerical summaries such as tables and bar graphs, comparing parts of the data.