INTRODUCTION

Becoming a lifelong learner and problem solver is rooted in wonder and curiosity. As an early childhood educator, you have a unique opportunity to nurture these traits in children by listening to the many questions they ask about the world, helping them explore what they notice in meaningful ways, and engaging them in rich conversations that spark even more discoveries, deeper understanding, and new wonderings!

Many of the things children naturally wonder about are bursting with math, as are the stories they read, the games they play, their day-to-day routines, and the real-life problems they face. You just have to look to find it: "The chart says it's my turn next, right?," "Will my new school be bigger than this school?," "Zak, we need two more long blocks for the wall. Can we use four medium ones instead?" Think about a book the children in your class love hearing read aloud, whether it's a fairytale, a classic children's book, or a new picture book. Is there a problem the character or characters must solve? Now look a little harder—how can you find the math in the problem situation?

When you have an eye for seeing the math all around us, you can guide children in finding it for themselves.

Building Math Minds

You can build on children's natural curiosity and observations to help them grow into confident problem solvers and investigators of math concepts. To do so, it's helpful to understand a few basic principles about how to make math both meaningful and enjoyable in early childhood—and beyond.

Learning Math Is Grounded in Conceptual Understanding

We are all born with the ability to do math, but like walking and talking, it takes a lot of practice! Unlike walking and talking, however, math is much more than a skill—it is a conceptual understanding that develops over time. You don't need to know the mechanics of how your brain and nervous system and muscles work together to walk across the room, but to do math, you do need to understand how numbers can be used to find out "how many" objects are in a collection or why it's important to specify what kind of "big" you're talking about before you can determine or compare an object's size.

When using words like . . .			To say things like . . .	You are talking about . . .	
bigger colder lots	loudest more saltier	tiny wide	"Do you want two more apple slices?" "The puppy is smaller than the big dog."	compare and contrast relationships	magnitude measurement quantity
alike different	the same similar		"TJ chose the same kind of van as you, but his is different because it's red and yours is white."	making sets sorting	
beneath down far away	near next to over	pointy round under	"I think the ball rolled under the table."	geometric attributes (shapes and lines) spatial relationships	

Engaging children in math talk while exploring stories, events, and interests that matter to them is an effective way to help them make sense of essential math concepts. No matter what a child's home language or culture is, mathematical concepts are already built into everyday speech and communication from early on. Some common examples are shown in the figure above.

Of course, depending on the context, words like those in the figure could be part of conversations about many other math concepts besides the examples named. Math concepts connect, overlap, and build on one another; they do not exist as isolated ideas.

Young children's use and understanding of mathematical language evolves over time. The more they hear math words and see them demonstrated in the context of their everyday experiences, the better developed their mathematical thinking skills become. For example, a toddler might call every animal with four legs he sees *dog*.

As important people in the child's life respond to his use of the word, including pointing out differences, the child processes this information and matches it to categories that already exist in his mind. Over time, he learns that while both a dog and a horse have four legs, a horse is something different from a dog. In the same way, children's understanding of mathematical concepts also becomes more precise. While a young child might be able to recite the words for the numbers 1 through 10, this does not mean that she understands that the word *ten* means there is a quantity of exactly 10 things. Teaching number words *and* number concepts together is essential to scaffolding understanding and open-ended thinking.

Be intentional about using language that brings out the math as you talk, interact, and play with children. As they experience mathematical ideas in many different contexts and gain an understanding of basic math language, children are ready to explore increasingly more complex mathematical concepts.

Math Is Everywhere

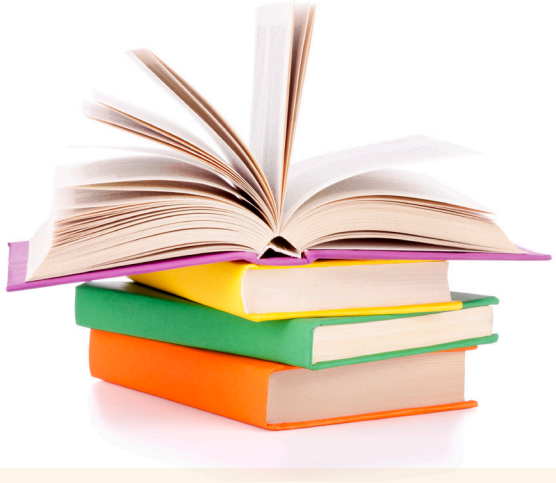
Many early childhood experts agree that mathematizing is a critical starting point to help children understand and be interested in math. *Mathematizing* means seeing math in daily life and using mathematical language and concepts to frame, analyze, and explore situations. You are mathematizing when you

- Engage children in talking about their ride on the bus this morning: "How many people were on the bus with you? How long did it take you to get to school this morning?"
- Ask a child to tell you more about the drawing she made of her home: "My bedroom is upstairs. It has two beds, one for me and one for my sister."
- Help children resolve real-life problems and conflicts: "Joey, I hear you saying it is not fair that you have two cars when Taahira has four. What could you both do about this? . . . Okay, Joey's idea is that he plays with all six cars for five minutes, and then Taahira plays with all six for five minutes. . . . You don't like that idea, Taahira? What if you line up all the cars together and each choose one at a time so you each have an equal number? Or maybe it would be more fun for you both to play with all six cars together?"

You can also develop intentional learning experiences and lesson plans around mathematizing. What are the children interested in? What kinds of things do you hear them talking about? What's going on in their families and communities? A mathematical inquiry is most meaningful for children when it is integrated with something they are already familiar with and interested in. This is true for everyone across all areas of learning—it's easier for us to grasp new ideas and ways of doing things when they are tied to things we already know. Because they are such an important element of every child's life, stories, games, and routines are powerful entry points to create and introduce high-quality mathematical experiences—even if they don't initially appear to be math focused.

> The children in Mr. Van's kindergarten class are doing an author study on Mo Willems. Although this is a literacy study, Mr. Van ties in math experiences to the children's interests. The reading center has a big collection of Willems's books, and the children are discussing how to organize them all. With Mr. Van's guidance, they decide to sort them by series: one group for all the books about Knuffle Bunny; one for Pigeon books; another group for books about Elephant and Piggie; and one for Elephant and Piggie Like Reading! books.

On a poster board, Garrett and Anika draw a chart that displays how many books are in each series set. Another display on the wall shows the results of a survey the children conducted as a whole group with Mr. Van to find out which books are everyone's favorites, how many children have the same favorite book, and which book is the biggest favorite.



Sakura, Gerald, and Liam decide to act out *We Are Growing*, an Elephant and Piggie Like Reading! book by Laurie Keller. Mr. Van observes them as they work out how many children are needed to play all the characters (blades of grass!) and how they will act out growing taller. The next day, Liam brings in a photo of the growth chart his parents have used to track his height since he was a baby. Excitedly, other children begin sharing and comparing the growth charts that their own families have at home. Mr. Van is delighted by all the math learning that is coming out of what appeared to be a literacy study!

Helpful Websites

Here are a few online resources that are full of ideas for how you can mathematize learning experiences in your classroom.

Development and Research in Early Math Education (DREME) Network: https://dreme.stanford.edu

Erikson Institute's Early Math Collaborative: https://earlymath.erikson.edu

Illuminations, by the National Council of Teachers of Mathematics (NCTM): https://illuminations.nctm.org

Math Is More than Just Getting the Right Answer—It's About Questions and Thinking

Math is so much more interesting and complex than rote counting and labeling, flash cards, or number problems on a worksheet. It is a useful, meaning-making way of thinking that is steeped in *logical-mathematical thinking*. This means using reasoning skills to

- Identify the problem or what you want to know: "What size wagon do I need to hold all of my blocks? Where should I put the wagon so the blocks are easy to get out and put away in the block center?"
- Find a solution by analyzing the situation and using cause-and-effect thinking to understand the relationships among objects, actions, or ideas: "How many blocks do I have? Does the wagon I need fit in the block center right now? How can I rearrange the room so I can bring the wagon closer?"

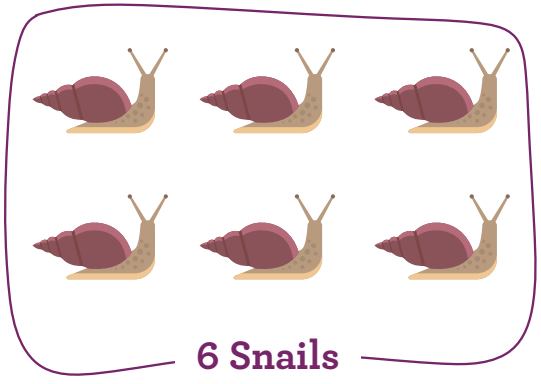
It is more important that children learn to be mathematical thinkers than it is that they can recite the right answer to a question or problem. Even in cases where there is a single correct answer, it is important to understand that there are different ways to arrive at that solution. Emphasize the process more than the solution, and cultivate this mindset in the children. This emphasis on process, thinking, and reasoning also ties in with the mathematical practices outlined by the Common Core State Standards (see "Standards for Mathematical Practice" on this page. When you value children's thinking and reasoning, you encourage them to think through situations themselves, try out a variety of strategies, and communicate their ideas and what they've learned. These are skills that apply to every part of their lives.

Standards for Mathematical Practice

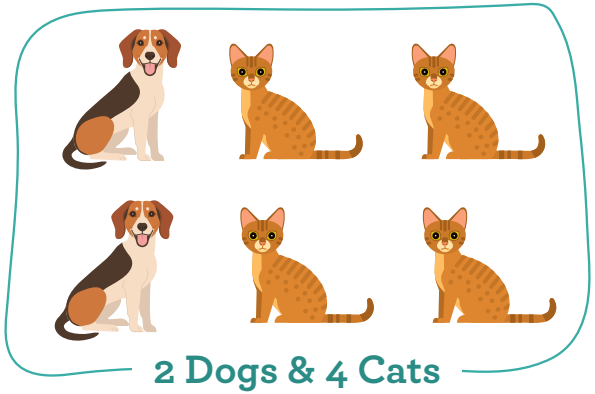
Children develop the problem-solving skills and processes they need to succeed when you provide them with math experiences that focus on the following mathematical practices outlined by the Common Core State Standards for Mathematics (NGA & CCSSO 2010):

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Young children are very concrete thinkers, and they need many hands-on experiences and opportunities to talk about what they're doing before they can generalize the mathematical concepts they are learning. For example, it is likely to take some time for a child be able to transfer the skill of counting animals in a picture to counting out and handing someone exactly three stuffed animals from a box. It will take even longer to understand the idea that a quantity can be put together (composed) and broken apart (decomposed) in many different ways. A group of six animals might be made up of six snails; two dogs and four cats; three foxes and three rabbits; or one turtle and five fish.

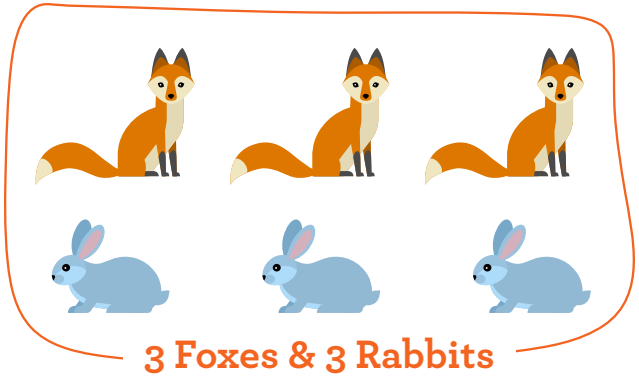


6 Snails



2 Dogs & 4 Cats

If we think of understanding as a lightbulb, the path to switching it on is different for each individual. As children work together, ask questions, and share their ideas, they hear and consider things they haven't thought of before and develop vital problem-solving skills like collaboration, communication, brainstorming, and flexible thinking. When you facilitate this process by using what each child knows and ask open-ended questions that inspire deeper thinking, children are better able to make the meaningful connections between what they know and what they want to know until that lightbulb switches on. The questions you ask and the conversations you start scaffold children's learning and encourage children to consider their thinking from a whole new angle, which helps children develop abstract thinking skills. As you find opportunities to invite children to explore and make discoveries through the questions you ask them, the more questions budding mathematicians will come up with themselves!



3 Foxes & 3 Rabbits

1 Turtle & 5 Fish

Math Is for Everyone

All children bring to the classroom their individual *funds of knowledge*—the body of knowledge, skills, and experiences they've built through interactions with their families and communities. This means children will see things differently and will use different methods to make sense of what is happening as they explore math. For example, a child who speaks both English and Chinese might name a quantity of 12 objects as "ten two." A teacher who has made efforts to learn more about this child's home language might realize that unlike English, Chinese has a regular number word system. That is to say, while the English language has unique words to name numbers larger than 10, the Chinese language combines the words for 1 through 10 to create larger number words. In English, the number word for 12 is "twelve" while in Chinese it is *shi er*, which literally translates to "ten two" (Rasmussen et al. 2006). With this understanding, the teacher can work to bridge the child's knowledge between both languages. Scenarios like this are why it is critical to respect and build on each child's funds of knowledge. Accepting only certain answers as right and dismissing the ways other children represent their understanding is wrong. Not only is this not what math is about, it's counterproductive.

One approach to making math learning more inclusive and equitable is to use rich storybooks as a springboard. Exploring math by examining problem situations in a story or in real life makes math approachable and invites children to engage with the problem and come up with a solution. Books that are most likely to spark mathematical thinking share several characteristics:

- A strong story line or theme
- A problem that children can think about logically
- Engaging text and illustrations
- Something new or interesting to think about every time you read it
- An invitation to explore and play with ideas in a meaningful way

Time and again, play has proven to be one of the most effective ways to build on and reinforce children's learning. Play invites children to be actively involved in problem solving not only cognitively but also physically, socially, and emotionally. Consider which question would get children more engaged in finding the solution to the problem: "How many dogs were chasing the Gingerbread Man?" or "How many of you need to pretend to be the dogs chasing the Gingerbread Man?" The first question is just looking for a set answer, while the second gets children emotionally invested because it is linked to a real-world situation: "I want to be a dog! One . . . two . . . three . . . three of us can be dogs!"

About the Book

This book focuses on five questions that children wonder about and ways you can guide children to investigate and understand these questions. Because this is a book about math, the chapters explore how to bring out the logical-mathematical thinking at the heart of these questions. The order of the chapters reflects the way children's mathematical thinking develops, from less complex to more complex and from concrete to abstract.

How Are These the Same? How Are These Different? (Matching and Sorting)

By the time they are in preschool, many children know how to put away books and toys, organize a collection of treasures, and create a birthday wish list. Everyday actions like these become mathematical when you intentionally guide children to focus on

- Identifying and naming key attributes of objects
- Using precise, or specific, language
- Making rules about what attributes all items in a set must share
- Comparing, contrasting, and ordering objects by size, type, color, or quantity

What Comes Next? (Patterns)

Children, like all humans, are always trying to understand, establish, or change patterns and structure. Young children need many repeated, guided experiences to develop the ability to identify and describe the patterns all around them, from the sequence of daily classroom routines to the motions of a finger play to the repeated words in a story. Identifying the structure of patterns allows them to predict what will come next—and children love to be proven right! As their thinking develops further, they become aware of more complex patterns that are rooted in math; for example, birthdays follow a plus-one rule and the number of monkeys jumping on the bed follows a minus-one rule.

How Many Do We Have, Need, or Want? (Number Sense)

From a young age, before they know and can use number words, children know when they want *more* or *less* of something. This is the beginning of understanding the mathematical concept of quantity. It can take a while before children are precise about how *many* more or fewer of something they need or want. They need plenty of time, experiences, and practice to explore counting and number operations before they can estimate or come up with an exact answer to the question of *how many?*

How Big Is It? (Measuring)

Young children tend to believe that bigger is always better. You can guide them toward using more logical thinking around the concept of size. To figure out how big something is means exploring more specific questions about *what kind* of big we are thinking about. Do we want to know how tall the boat is (height)? How long it is (length)? How wide it is (width)? How old it is (age)? How heavy it is (weight)? How many people it can hold (capacity)? Understanding the question of how big something is also means offering children lots of measuring experiences that have them explore

- Understanding when a general sense of size is good enough (estimation) and when a more precise measurement is needed
- Using direct and indirect comparisons
- Using standard and nonstandard units of measurement

Where Is It? (Spatial Relationships)

Most children have had the experience of trying to find someone (a friend on the playground, a family member in the house) or something (a toy in their bedroom, a book in the reading center). To find what they are looking for, they might remember that the person or thing is located in relation to someone or something else (their friend is playing by the swings or their stuffed penguin is in a basket on the shelf). With repeated experiences like these, they come to realize that following or giving directions requires logical thinking and precise use of language. For example, being told that a favorite toy is *somewhere* isn't helpful, but it only takes a minute to find it when told "I saw it under the yellow chair in the back of the room."

Each chapter of this book introduces mathematical problem situations that explore one of these questions. To make the investigation of each of these questions meaningful for young children, they are embedded in children's books, games, and routines and include lots of opportunities for conversation. These mathematical explorations are most appropriate for children in preschool and kindergarten, and they are flexible enough that slight adjustments can be made to make them more or less complex as needed for diverse learners. Each chapter also includes a list of math concepts supported by the activities in it.

The practical activities and ideas in this book link *math* with *questions* and *wonder*. This mindset may be quite different from your own experiences in school, where it might have been drilled into you that right answers and speed are more important than understanding. By embedding investigations in stories and real-life problem situations for children and engaging them in intentional, thoughtful conversations about math, you are laying the critical groundwork for later mathematics.

As with any resource book, begin at the place that makes sense for the children you teach. You know your curriculum and the strengths and needs of the children in your classroom best, so it's up to you to choose and tailor experiences that are a good fit for them. Reflect on what children learn from an experience, what they express interest in, and how you can build on their growing understanding to extend the concepts. If some children seem to need more time to explore a concept, repeat an activity or change it up.

As you explore these stories and activities with the children, we hope that all of you will experience the wonder that is math!